



Lecture 10 of 42

Logical Agents and Propositional Logic Discussion: Logic in AI

Friday, 14 September 2007

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KSOL course page: <http://snipurl.com/v9v3>

Course web site: <http://www.kddresearch.org/Courses/Fall-2007/CIS730>

Instructor home page: <http://www.cis.ksu.edu/~bhsu>

Reading for Next Class:

Section 7.5 – 7.7, p. 211 - 232, Russell & Norvig 2nd edition



Lecture Outline

- **Reading for Next Class: Sections 7.5 – 7.7, R&N 2^e**
- **Today: Logical Agents**
 - * Classical knowledge representation
 - * Limitations of the classical symbolic approach
 - * Modern approach: representation, reasoning, learning
 - * “New” aspects: uncertainty, abstraction, classification paradigm
- **Next Week: Start of Material on Logic**
 - * Representation: “a bridge between learning and reasoning” (Koller)
 - * Basis for automated reasoning: theorem proving, other inference





Overview

- **Today's Reading**
 - * Sections 7.1 – 7.4, Russell and Norvig 2e
 - * Recommended references: Nilsson and Genesereth (*Logical Foundations of AI*)
- **Previously: Logical Agents**
 - * Knowledge Bases (KB) and KB agents
 - * Motivating example: Wumpus World
 - * Logic in general
 - * Syntax of propositional calculus
- **Today**
 - * Propositional calculus (concluded)
 - * Normal forms
 - * Production systems
 - * Predicate logic
 - * Introduction to First-Order Logic (FOL): examples, inference rules (sketch)
- **Next Week: First-Order Logic Review, Resolution**



Knowledge Representation (KR) for Intelligent Agent Problems

- Percepts
 - * *What can agent observe?*
 - * *What can sensors tell it?*
- Actions
 - * *What actuators does agent have?*
 - * *In what context are they applicable?*
- Goals
 - * *What are agents goals? Preferences (utilities)?*
 - * *How does agent evaluate them (check environment, deliberate, etc.)?*
- Environment
 - * *What are "rules of the world"?*
 - * *How can these be represented, simulated?*



Review: Simple Knowledge-Based Agent

```

function KB-AGENT(percept) returns an action
static: KB, a knowledge base
         t, a counter, initially 0, indicating time
  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
  
```

The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Adapted from slides by S. Russell, UC Berkeley

Figure 6.1 p. 152 R&N



Review: Types of Logic

Logics are characterized by what they commit to as "primitives"

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0...1
Fuzzy logic	degree of truth	degree of belief 0...1

Adapted from slides by S. Russell, UC Berkeley

Figure 6.7 p. 166 R&N





Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $A \quad B \quad C$
True True False

Rules for evaluating truth with respect to a model m :

$\neg S$ is true iff S is false
 $S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true
 $S_1 \vee S_2$ is true iff S_1 is true or S_2 is true
 $S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true
 i.e., is false iff S_1 is true and S_2 is false
 $S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

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Propositional Inference: Enumeration (Model Checking) Method

Let $\alpha = A \vee B$ and $KB = (A \vee C) \wedge (B \vee \neg C)$

Is it the case that $KB \models \alpha$?

Check all possible models— α must be true wherever KB is true

A	B	C	$A \vee C$	$B \vee \neg C$	KB	α
<i>False</i>	<i>False</i>	<i>False</i>	<i>False</i>	<i>True</i>	<i>False</i>	<i>False</i>
<i>False</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>False</i>	<i>False</i>	<i>False</i>
<i>False</i>	<i>True</i>	<i>False</i>	<i>False</i>	<i>True</i>	<i>False</i>	<i>True</i>
<i>False</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>
<i>True</i>	<i>False</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>
<i>True</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>False</i>	<i>False</i>	<i>True</i>
<i>True</i>	<i>True</i>	<i>False</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>
<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>True</i>

Adapted from slides by S. Russell, UC Berkeley



Normal Forms: CNF, DNF, Horn

Other approaches to inference use syntactic operations on sentences, often expressed in standardized forms

Conjunctive Normal Form (CNF—universal)
conjunction of disjunctions of literals
clauses

$$\text{E.g., } (A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$$

Disjunctive Normal Form (DNF—universal)
disjunction of conjunctions of literals
terms

$$\text{E.g., } (A \wedge B) \vee (A \wedge \neg C) \vee (A \wedge \neg D) \vee (\neg B \wedge \neg C) \vee (\neg B \wedge \neg D)$$

Horn Form (restricted)
conjunction of Horn clauses (clauses with ≤ 1 positive literal)

$$\text{E.g., } (A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$$

Often written as set of implications:

$$B \Rightarrow A \text{ and } (C \wedge D) \Rightarrow B$$

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Validity and Satisfiability

A sentence is valid if it is true in all models

$$\text{e.g., } A \vee \neg A, \quad A \Rightarrow A, \quad (A \wedge (A \Rightarrow B)) \Rightarrow B$$

Validity is connected to inference via the Deduction Theorem:

$$KB \models \alpha \text{ if and only if } (KB \Rightarrow \alpha) \text{ is valid}$$

A sentence is satisfiable if it is true in some model

$$\text{e.g., } A \vee B, \quad C$$

A sentence is unsatisfiable if it is true in no models

$$\text{e.g., } A \wedge \neg A$$

Satisfiability is connected to inference via the following:

$$KB \models \alpha \text{ if and only if } (KB \wedge \neg \alpha) \text{ is unsatisfiable}$$

i.e., prove α by *reductio ad absurdum*

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Proof Methods

Proof methods divide into (roughly) two kinds:

Model checking

truth table enumeration (sound and complete for propositional)
heuristic search in model space (sound but incomplete)
e.g., the GSAT algorithm (Ex. 6.15)

Application of inference rules

Legitimate (sound) generation of new sentences from old

Proof = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

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Inference (Sequent) Rules for Propositional Logic

Resolution (for CNF): complete for propositional logic

$$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \quad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining

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Logical Agents: Taking Stock

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks

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The Road Ahead: Predicate Logic and FOL

- Predicate Logic
 - * Enriching language
 - ⇒ Predicates
 - ⇒ Functions
 - * Syntax and semantics of predicate logic
- First-Order Logic (FOL, FOFC)
 - * Need for quantifiers
 - * Relation to (unquantified) predicate logic
 - * Syntax and semantics of FOL
- Fun with Sentences
- Wumpus World in FOL

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Syntax of FOL: Basic Elements

Constants *KingJohn, 2, UCB, ...*
Predicates *Brother, >, ...*
Functions *Sqrt, LeftLegOf, ...*
Variables *x, y, a, b, ...*
Connectives $\wedge \vee \neg \Rightarrow \Leftrightarrow$
Equality $=$
Quantifiers $\forall \exists$

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FOL: Atomic Sentences (Atomic Well-Formed Formulae)

Atomic sentence = $predicate(term_1, \dots, term_n)$
or $term_1 = term_2$

Term = $function(term_1, \dots, term_n)$
or *constant* or *variable*

E.g., $Brother(KingJohn, RichardTheLionheart)$
 $> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Adapted from slides by S. Russell, UC Berkeley



Summary Points

- Logical Agents Overview (Last Time)
 - * Knowledge Bases (KB) and KB agents
 - * Motivating example: Wumpus World
 - * Logic in general
 - * Syntax of propositional calculus
- Propositional and First-Order Calculi (Today)
 - * Propositional calculus (concluded)
 - ⇒ Normal forms
 - ⇒ Inference (aka sequent) rules
 - * Production systems
 - * Predicate logic without quantifiers
 - * Introduction to First-Order Logic (FOL)
 - ⇒ Examples
 - ⇒ Inference rules (sketch)
- Next Week: First-Order Logic Review, Intro to Resolution Theorem

Proving



Fun with Sentences: Family Feud

- Brothers are Siblings
 - * $\forall x, y . \text{Brother}(x, y) \Leftrightarrow \text{Sibling}(x, y)$
- Siblings (i.e., Sibling Relationships) are Reflexive
 - * $\forall x, y . \text{Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$
- One's Mother is One's Female Parent
 - * $\forall x, y . \text{Mother}(x, y) \Leftrightarrow \text{Female}(x) \wedge \text{Parent}(x, y)$
- A First Cousin Is A Child of A Parent's Sibling
 - * $\forall x, y . \text{First-Cousin}(x, y) \Leftrightarrow$
 $\exists p, ps . \text{Parent}(p, x) \wedge \text{Sibling}(p, ps) \wedge \text{Parent}(ps, y)$

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Jigsaw Exercise [1]: First-Order Logic Sentences

- “Every Dog Chases Its Own Tail”
 - * $\forall d . \text{Chases}(d, \text{tail-of}(d))$
 - * **Alternative Statement:** $\forall d . \exists t . \text{Tail-Of}(t, d) \wedge \text{Chases}(d, t)$
 - * **Prefigures concept of Skolemization** (Skolem variables / functions)
- “Every Dog Chases Its Own (Unique) Tail”
 - * $\forall d . \exists^1 t . \text{Tail-Of}(t, d) \wedge \text{Chases}(d, t) \equiv$ $\forall d .$
 $\exists t . \text{Tail-Of}(t, d) \wedge \text{Chases}(d, t) \wedge [\forall t' \text{Chases}(d, t') \Rightarrow t' = t]$
- “Only The Wicked Flee when No One Pursueth”
 - * $\forall x . \text{Flees}(x) \wedge [\neg \exists y \text{Pursues}(y, x)] \Rightarrow \text{Wicked}(x)$
 - * **Alternative :** $\forall x . [\exists y . \text{Flees}(x, y)] \wedge [\neg \exists z . \text{Pursues}(z, x)] \Rightarrow \text{Wicked}(x)$
- **Offline Exercise: What Is An n th Cousin, m Times Removed?**



Jigsaw Exercise [2]: First-Order Logic Sentences





Terminology

- **Logical Frameworks**
 - * Knowledge Bases (KB)
 - * Logic in general: representation languages, syntax, semantics
 - * Propositional logic
 - * First-order logic (FOL, FOFC)
 - * Model theory, domain theory: possible worlds semantics, entailment
- **Normal Forms**
 - * Conjunctive Normal Form (CNF)
 - * Disjunctive Normal Form (DNF)
 - * Horn Form
- **Proof Theory and Inference Systems**
 - * Sequent calculi: rules of proof theory
 - * Derivability or provability
 - * Properties
 - ⇒ Soundness (derivability implies entailment)
 - ⇒ Completeness (entailment implies derivability)

