



## Lecture 13 of 42

### First-Order Logic: KE and Theorem Proving Discussion: Review of Models, Theorem Proving

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KSOL course page: <http://snipurl.com/v9v3>

Course web site: <http://www.kddresearch.org/Courses/Fall-2007/CIS730>

Instructor home page: <http://www.cis.ksu.edu/~bhsu>

#### Reading for Next Class:

Section 8.3 – 8.5, p. 240 – 268, Russell & Norvig 2<sup>nd</sup> edition

Section 9.1, p. 272 – 275, Russell & Norvig 2<sup>nd</sup> edition



## Lecture Outline

- Reading for Next Class: Section 8.1 – 8.2, R&N 2e
- Recommended : Nilsson and Genesereth (Chapter 5 online)
- Next Week's: Chapter 8 & first half of Chapter 9, R&N
- Today: Knowledge Engineering and Theorem Proving
- Next Week
  - \* Resolution
  - \* Constraint logic
  - \* Prolog
- Week of 04 Oct 2006
  - \* Knowledge representation
  - \* Ontologies





## Logical Agents: Review

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks

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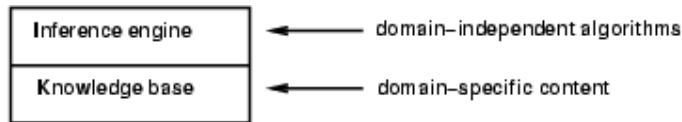
## Predicate Logic and FOL Road Map

- **Predicate Logic**
  - \* **Enriching language**
    - ⇒ **Predicates**
    - ⇒ **Functions**
  - \* **Syntax and semantics of predicate logic**
- **First-Order Logic (FOL, FOPL)**
  - \* **Need for quantifiers**
  - \* **Relation to (unquantified) predicate logic**
  - \* **Syntax and semantics of FOL**
- **Fun with Sentences**
- **Wumpus World in FOL**





## Knowledge bases: Review



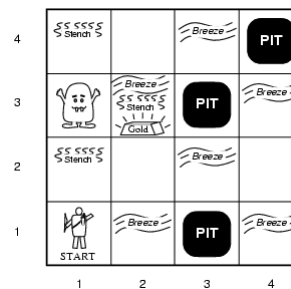
- Knowledge base = set of **sentences** in a **formal language**
- **Declarative** approach to building an agent (or other system):
  - \* **Tell it what it needs to know**
- Then it can **Ask** itself what to do - answers should follow from **KB**
- Agents can be viewed at the **knowledge level**
  - i.e., what they know, regardless of how implemented
- Or at the **implementation level**
  - \* i.e., data structures in KB and algorithms that manipulate them

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## Wumpus World – PEAS Description: Review

- **Performance measure**
  - \* gold +1000, death -1000
  - \* -1 per step, -10 for using the arrow
- **Environment**
  - \* Squares adjacent to wumpus are smelly
  - \* Squares adjacent to pit are breezy
  - \* Glitter iff gold is in the same square
  - \* Shooting kills wumpus if you are facing it
  - \* Shooting uses up the only arrow
  - \* Grabbing picks up gold if in same square
  - \* Releasing drops the gold in same square



- **Sensors:** Stench, Breeze, Glitter, Bump, Scream
- **Actuators:** Left turn, Right turn, Forward, Grab, Release, Shoot

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## Wumpus world – characterization: Review

- **Fully Observable** No – only **local** perception
- **Deterministic** Yes – outcomes exactly specified
- **Episodic** No – sequential at the level of actions
- **Static** Yes – Wumpus and Pits do not move
- **Discrete** Yes
- **Single-agent?** Yes – Wumpus is essentially a natural feature

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## Logic in General: Review

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the “meaning” of sentences;  
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$  is a sentence;  $x^2 + y >$  is not a sentence

$x + 2 \geq y$  is true iff the number  $x + 2$  is no less than the number  $y$

$x + 2 \geq y$  is true in a world where  $x = 7, y = 1$

$x + 2 \geq y$  is false in a world where  $x = 0, y = 6$

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## Entailment: Review

- **Entailment** means that one thing **follows from** another:

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence  $\alpha$  if and only if  $\alpha$  is true in all worlds where *KB* is true

- \* E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”
- \* E.g.,  $x+y = 4$  entails  $4 = x+y$
- \* Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

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## DPLL algorithm: Review

**function** DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

**inputs:** *s*, a sentence in propositional logic

*clauses* ← the set of clauses in the CNF representation of *s*

*symbols* ← a list of the proposition symbols in *s*

**return** DPLL(*clauses*, *symbols*, [])

**function** DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

**if** every clause in *clauses* is true in *model* **then return** *true*

**if** some clause in *clauses* is false in *model* **then return** *false*

*P*, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols*-*P*, [*P* = *value* | *model*])

*P*, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols*-*P*, [*P* = *value* | *model*])

*P* ← FIRST(*symbols*); *rest* ← REST(*symbols*)

**return** DPLL(*clauses*, *rest*, [*P* = *true* | *model*]) **or**

DPLL(*clauses*, *rest*, [*P* = *false* | *model*])

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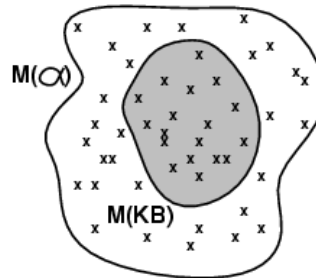
Figure 7.16 p. 222 R&N 2e





## Models: Review

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say  $m$  is a **model** of a sentence  $\alpha$  if  $\alpha$  is true in  $m$
- $M(\alpha)$  is the set of all models of  $\alpha$
- Then  $KB \vdash \alpha$  iff  $M(KB) \subseteq M(\alpha)$ 
  - \* E.g.  $KB =$  Giants won and Reds won  $\alpha =$  Giants won
- See: definitions on p. 201, 203 R&N 2e

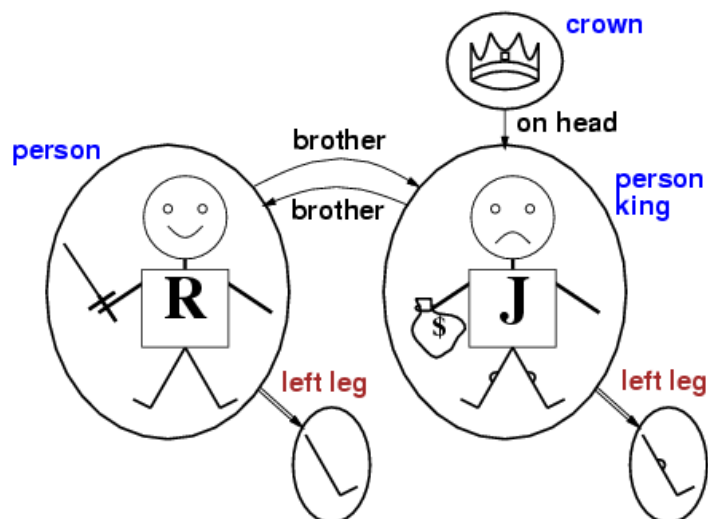


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See also: S. 7.3, p. 200 – 204  
S. 8.2, p. 245 – 253



## Models for FOL: Example





## Types of Logic: Review

Logics are characterized by what they commit to as “primitives”

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0..1
Fuzzy logic	degree of truth	degree of belief 0..1

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Figure 8.1 p. 244 R&N 2e



## FOL – Atomic Sentences (Atoms): Review

Atomic sentence =  $predicate(term_1, \dots, term_n)$   
or  $term_1 = term_2$

Term =  $function(term_1, \dots, term_n)$   
or *constant* or *variable*

E.g.,  $Brother(KingJohn, RichardTheLionheart)$   
>  $(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn))$

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## FOL – Complex Sentences (WFFs): Review

Complex sentences are made from atomic sentences using connectives

$$\neg S, S_1 \wedge S_2, S_1 \vee S_2, S_1 \Rightarrow S_2, S_1 \Leftrightarrow S_2$$

E.g.  $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$

$$>(1, 2) \vee \leq(1, 2)$$

$$>(1, 2) \wedge \neg >(1, 2)$$

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## Truth in FOL: Review

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for

*constant symbols* → objects

*predicate symbols* → relations

*function symbols* → functional relations

An atomic sentence  $predicate(term_1, \dots, term_n)$  is true  
iff the objects referred to by  $term_1, \dots, term_n$   
are in the relation referred to by *predicate*

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## Automated Deduction (Chapters 8-10): Review

Sound inference: find  $\alpha$  such that  $KB \models \alpha$ .

Proof process is a search, operators are inference rules.

E.g., Modus Ponens (MP)

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta} \quad \frac{At(Joe, UCB) \quad At(Joe, UCB) \Rightarrow OK(Joe)}{OK(Joe)}$$

E.g., And-Introduction (AI)

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \quad \frac{OK(Joe) \quad CSMajor(Joe)}{OK(Joe) \wedge CSMajor(Joe)}$$

E.g., Universal Elimination (UE)

$$\frac{\forall x \alpha}{\alpha\{x/\tau\}} \quad \frac{\forall x At(x, UCB) \Rightarrow OK(x)}{At(Pat, UCB) \Rightarrow OK(Pat)}$$

$\tau$  must be a ground term (i.e., no variables)

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## Example Proof

- |  |  |
|--|--|
| <ul style="list-style-type: none"> <li>● Bob is a buffalo</li> <li>● Pat is a pig</li> <li>● Buffaloes outrun pigs</li> <li>● Bob outruns Pat</li> </ul> | <ol style="list-style-type: none"> <li>1. <i>Buffalo</i>(Bob)</li> <li>2. <i>Pig</i>(Pat)</li> <li>3. <math>\forall x, y \text{ Buffalo}(x) \wedge \text{Pig}(y) \Rightarrow \text{Faster}(x, y)</math></li> </ol> |
|--|--|

- Apply Sequent Rules to Generate New Assertions

- |  |   |
|--|---|
| <p>AI 1 &amp; 2</p> <p>UE 3, <math>\{x/Bob, y/Pat\}</math></p> <p>MP 6 &amp; 7</p> | <ol style="list-style-type: none"> <li>4. <i>Buffalo</i>(Bob) <math>\wedge</math> <i>Pig</i>(Pat)</li> <li>5. <i>Buffalo</i>(Bob) <math>\wedge</math> <i>Pig</i>(Pat) <math>\Rightarrow</math> <i>Faster</i>(Bob, Pat)</li> <li>6. <i>Faster</i>(Bob, Pat)</li> </ol> |
|--|---|

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta}$$

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta}$$

$$\frac{\forall x \alpha}{\alpha\{x/\tau\}}$$

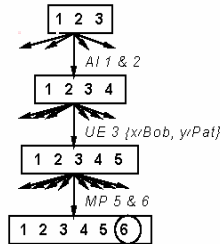
- Modus Ponens
  - And Introduction
  - Universal Elimination
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## Search with Primitive Inference Rules

Operators are inference rules  
 States are sets of sentences  
 Goal test checks state to see if it contains query sentence



AI, UE, MP is a common inference pattern

Problem: branching factor huge, esp. for UE

Idea: find a substitution that makes the rule premise match some known facts  
 $\Rightarrow$  a single, more powerful inference rule

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## A Brief History of Reasoning: Chapter 8 End Notes, R&N

450B.C.	Stoics	propositional logic, inference (maybe)
322B.C.	Aristotle	"syllogisms" (inference rules), quantifiers
1565	Cardano	probability theory (propositional logic + uncertainty)
1847	Boole	propositional logic (again)
1879	Frege	first-order logic
1922	Wittgenstein	proof by truth tables
1930	Gödel	$\exists$ complete algorithm for FOL
1930	Herbrand	complete algorithm for FOL (reduce to propositional)
1931	Gödel	$\neg\exists$ complete algorithm for arithmetic
1960	Davis/Putnam	"practical" algorithm for propositional logic
1965	Robinson	"practical" algorithm for FOL—resolution

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## Knowledge Engineering

- **KE: Process of**
  - \* Choosing logical language (basis of KR)
  - \* Building KB
  - \* Implementing proof theory
  - \* Inferring new facts
- **Analogy: Programming Languages / Software Engineering**
  - \* Choosing programming language (basis of software engineering)
  - \* Writing program
  - \* Choosing / writing compiler
  - \* Running program
- **Example Domains**
  - \* Electronic circuits (Section 8.3 R&N)
  - \* Exercise
    - ⇒ Look up, read about [protocol analysis](#)
    - ⇒ Find example and think about KE process for your project domain



## Unification: Definitions and Idea Sketch

A substitution  $\sigma$  unifies atomic sentences  $p$  and  $q$  if  $p\sigma = q\sigma$

$p$	$q$	$\sigma$
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/John, y/OJ\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$

**Idea:** Unify rule premises with known facts, apply unifier to conclusion

E.g., if we know  $q$  and  $Knows(John, x) \Rightarrow Likes(John, x)$   
 then we conclude  $Likes(John, Jane)$   
 $Likes(John, OJ)$   
 $Likes(John, Mother(John))$

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## Generalized Modus Ponens

$$\frac{p_1', p_2', \dots, p_n', (p_1 \wedge p_2 \wedge \dots \wedge p_n \Rightarrow q)}{q\sigma} \quad \text{where } p_i'\sigma = p_i\sigma \text{ for all } i$$

E.g.  $p_1' = \text{Faster}(\text{Bob}, \text{Fat})$   
 $p_2' = \text{Faster}(\text{Pat}, \text{Steve})$   
 $p_1 \wedge p_2 \Rightarrow q = \text{Faster}(x, y) \wedge \text{Faster}(y, z) \Rightarrow \text{Faster}(x, z)$   
 $\sigma = \{x/\text{Bob}, y/\text{Pat}, z/\text{Steve}\}$   
 $q\sigma = \text{Faster}(\text{Bob}, \text{Steve})$

GMP used with KB of definite clauses (*exactly one positive literal*):  
 either a single atomic sentence or  
 (conjunction of atomic sentences)  $\Rightarrow$  (atomic sentence)  
 All variables assumed universally quantified

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## Soundness of GMP

Need to show that

$$p_1', \dots, p_n', (p_1 \wedge \dots \wedge p_n \Rightarrow q) \models q\sigma$$

provided that  $p_i'\sigma = p_i\sigma$  for all  $i$

Lemma: For any definite clause  $p$ , we have  $p \models p\sigma$  by UE

1.  $(p_1 \wedge \dots \wedge p_n \Rightarrow q) \models (p_1 \wedge \dots \wedge p_n \Rightarrow q)\sigma = (p_1\sigma \wedge \dots \wedge p_n\sigma \Rightarrow q\sigma)$
2.  $p_1', \dots, p_n' \models p_1' \wedge \dots \wedge p_n' \models p_1'\sigma \wedge \dots \wedge p_n'\sigma$
3. From 1 and 2,  $q\sigma$  follows by simple MP

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## Forward Chaining

When a new fact  $p$  is added to the KB  
 for each rule such that  $p$  unifies with a premise  
 if the other premises are known  
 then add the conclusion to the KB and continue chaining

Forward chaining is data-driven  
 e.g., inferring properties and categories from percepts

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## Example: Forward Chaining

Add facts 1, 2, 3, 4, 5, 7 in turn.  
 Number in  $\square$  = unification literal;  $\surd$  indicates rule firing

1.  $Buffalo(x) \wedge Pig(y) \Rightarrow Faster(x, y)$
2.  $Pig(y) \wedge Slug(z) \Rightarrow Faster(y, z)$
3.  $Faster(x, y) \wedge Faster(y, z) \Rightarrow Faster(x, z)$
4.  $Buffalo(Bob) \square_{1a, \times}$
5.  $Pig(Pat) \square_{1b, \surd} \rightarrow \square_{2a, \times} \rightarrow \square_{3a, \times}, \square_{3b, \times}$
7.  $Slug(Steve) \square_{2b, \surd}$   
 $\rightarrow \square_{3a, \times}, \square_{3b, \surd}$   
 $\rightarrow \square_{3a, \times}, \square_{3b, \times}$

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## Backward Chaining

When a query  $q$  is asked  
 if a matching fact  $q'$  is known, return the unifier  
 for each rule whose consequent  $q'$  matches  $q$   
 attempt to prove each premise of the rule by backward chaining

(Some added complications in keeping track of the unifiers)

(More complications help to avoid infinite loops)

Two versions: find any solution, find all solutions

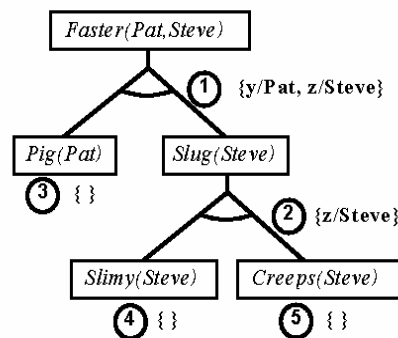
Backward chaining is the basis for logic programming, e.g., Prolog

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## Example: Backward Chaining

1.  $Pig(y) \wedge Slug(z) \Rightarrow Faster(y, z)$
2.  $Slimy(z) \wedge Creeps(z) \Rightarrow Slug(z)$
3.  $Pig(Pat)$       4.  $Slimy(Steve)$       5.  $Creeps(Steve)$



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## Backward Chaining

When a query  $q$  is asked  
 if a matching fact  $q'$  is known, return the unifier  
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 attempt to prove each premise of the rule by backward chaining

(Some added complications in keeping track of the unifiers)

(More complications help to avoid infinite loops)

Two versions: find any solution, find all solutions

Backward chaining is the basis for logic programming, e.g., Prolog

- Answer
  - \* Suppose ¬Query, For The Sake Of Contradiction (FTSOC)
  - \* Attempt to prove that  $KB \wedge \neg Query \vdash \perp$



## Resolution Inference Rule

Basic propositional version:

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta, \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

Full first-order version:

$$\frac{p_1 \vee \dots \vee p_j \dots \vee p_m, \quad q_1 \vee \dots \vee q_k \dots \vee q_n}{(p_1 \vee \dots \vee p_{j-1} \vee p_{j+1} \dots \vee p_m \vee q_1 \dots \vee q_{k-1} \vee q_{k+1} \dots \vee q_n)\sigma}$$

where  $p_j\sigma = \neg q_k\sigma$

For example,

$$\frac{\neg Rich(x) \vee Unhappy(x), \quad Rich(Me)}{Unhappy(Me)}$$

with  $\sigma = \{x/Me\}$

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## Digression: Decidability and Formal Languages

- See: Hopcroft and Ullman 2e, Lewis and Papadimitriou 3e
- Formal Languages (See: CIS 540, Other Automata Theory Course)
  - \* Member of Turing hierarchy
    - ⇒ Finite state automata: regular languages
    - ⇒ Pushdown automata: context-free languages
    - ⇒ Linear bounded automata: context-sensitive languages
    - ⇒ Turing machines: recursive languages
  - \* Recursive languages
    - ⇒  $\exists$  computational model for decision problem, halts in finite number of steps
    - ⇒ REC: set of all recursive languages
    - ⇒ Example: finite searches (convert to decision problem: *checking solution*)
    - ⇒ *Closed under complementation* (consequence?)
  - \* Recursive enumerable but not recursive (RE - REC)
  - \* Not recursive ( $\not\in$  RE)
- What Are FOL-VALID, FOL-NOT-SAT, FOL-SAT, FOL-NOT-VALID?



## Universal quantification

- $\forall \langle \text{variables} \rangle \langle \text{sentence} \rangle$
- Everyone at K-State is smart:
- $\forall x \text{ At}(x, \text{K-State}) \Rightarrow \text{Smart}(x)$
- $\forall x P$  is true in a model  $m$  iff  $P$  is true with  $x$  being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of  $P$ 
  - ⇒  $\text{At}(\text{KingJohn}, \text{K-State}) \Rightarrow \text{Smart}(\text{KingJohn})$
  - ⇒  $\wedge \text{At}(\text{Richard}, \text{K-State}) \Rightarrow \text{Smart}(\text{Richard})$
  - ⇒  $\wedge \text{At}(\text{K-State}, \text{K-State}) \Rightarrow \text{Smart}(\text{K-State})$
  - ⇒  $\wedge \dots$





## A common mistake to avoid

- Typically,  $\Rightarrow$  is the main connective with  $\forall$
- Common mistake: using  $\wedge$  as the main connective with  $\forall$ :  
 $\forall x \text{ At}(x, \text{K-State}) \wedge \text{Smart}(x)$   
means “Everyone is at K-State and everyone is smart”



## Existential quantification

- $\exists \langle \text{variables} \rangle \langle \text{sentence} \rangle$
- Someone at KU is smart:  
 $\exists x \text{ At}(x, \text{KU}) \wedge \text{Smart}(x)$
- $\exists x P$  is true in a model  $m$  iff  $P$  is true with  $x$  being some possible object in the model
- Roughly speaking, equivalent to the **disjunction** of **instantiations** of  $P$ 
  - ✓  $\text{At}(\text{KingJohn}, \text{KU}) \wedge \text{Smart}(\text{KingJohn})$
  - ✓  $\text{At}(\text{Richard}, \text{KU}) \wedge \text{Smart}(\text{Richard})$
  - ✓  $\text{At}(\text{KU}, \text{KU}) \wedge \text{Smart}(\text{KU})$
  - ✓ ...





## Summary Points

- Applications of Knowledge Bases (KBs) and Inference Systems
- “Industrial Strength” KBs
  - \* Building KBs
  - \* Components
    - ⇒ Ontologies
    - ⇒ Fact and rule bases
    - ⇒ Knowledge Engineering (KE) and protocol analysis
    - ⇒ Inductive Logic Programming (ILP) and other machine learning techniques
  - \* Using KBs
- Systems of Sequent Rules: GMP/AI/UE, Resolution
- Methodology of Inference
  - \* Inference as search
  - \* Forward and backward chaining
  - \* Fan-in, fan-out



## Terminology

- Logical Frameworks
  - \* Knowledge Bases (KB)
  - \* Logic in general: representation languages, syntax, semantics
  - \* Propositional logic
  - \* First-order logic (FOL, FOPL)
  - \* Model theory, domain theory: possible worlds semantics, entailment
- Normal Forms
  - \* Conjunctive Normal Form (CNF)
  - \* Disjunctive Normal Form (DNF)
  - \* Horn Form
- Proof Theory and Inference Systems
  - \* Sequent calculi: rules of proof theory
  - \* Derivability or provability
  - \* Properties
    - ⇒ Soundness (derivability implies entailment)
    - ⇒ Completeness (entailment implies derivability)