



Lecture 28 of 42

Graphical Models of Probability 1 Discussion: Representing Causality

Monday, 29 October 2007

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KSOL course page: <http://snipurl.com/v9v3>
Course web site: <http://www.kddresearch.org/Courses/Fall-2007/CIS730>
Instructor home page: <http://www.cis.ksu.edu/~bhsu>

Reading for Next Class:
Sections 14.1 – 14.2, Russell & Norvig 2nd edition



Lecture Outline

- **Today's Reading:** Chapter 13, Sections 14.1 – 14.2, R&N 2e
- **Wednesday's Reading:**
- **Today: Graphical models**
 - * Bayesian networks and causality
 - * Inference and learning
 - * BNJ interface (<http://bnj.sourceforge.net>)
 - * Causality





Bayes's Theorem: Review

- **Theorem**

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)} = \frac{P(h \wedge D)}{P(D)}$$

- **$P(h) \equiv$ Prior Probability of Assertion (Hypothesis) h**
 - * Measures initial beliefs (BK) before any information is obtained (hence prior)
- **$P(D) \equiv$ Prior Probability of Data (Observations) D**
 - * Measures probability of obtaining sample D (i.e., expresses D)
- **$P(h | D) \equiv$ Probability of h Given D**
 - * $|$ denotes conditioning - hence $P(h | D)$ is a conditional (aka posterior) probability
- **$P(D | h) \equiv$ Probability of D Given h**
 - * Measures probability of observing D given that h is correct (“generative” model)
- **$P(h \wedge D) \equiv$ Joint Probability of h and D**
 - * Measures probability of observing D and of h being correct



Bayesian Inference: Query Answering (QA)

- **Answering User Queries**
 - * Suppose we want to perform intelligent inferences over a database DB
 - ⇒ Scenario 1: DB contains records (instances), some “labeled” with answers
 - ⇒ Scenario 2: DB contains probabilities (annotations) over propositions
 - * **QA: an application of probabilistic inference**
- **QA Using Prior and Conditional Probabilities: Example**
 - * **Query: Does patient have cancer or not?**
 - * **Suppose: patient takes a lab test and result comes back positive**
 - ⇒ Correct + result in only 98% of cases in which disease is actually present
 - ⇒ Correct - result in only 97% of cases in which disease is not present
 - ⇒ Only 0.008 of the entire population has this cancer
 - * $\alpha \equiv P(\text{false negative for } H_0 \equiv \text{Cancer}) = 0.02$ (NB: for 1-point sample)
 - * $\beta \equiv P(\text{false positive for } H_0 \equiv \text{Cancer}) = 0.03$ (NB: for 1-point sample)
 - * $P(\text{Cancer}) = 0.008$ $P(+ | \text{Cancer}) = 0.98$ $P(+ | \neg \text{Cancer}) = 0.03$
 $P(\neg \text{Cancer}) = 0.992$ $P(- | \text{Cancer}) = 0.02$ $P(- | \neg \text{Cancer}) = 0.97$
 - * $P(+ | H_0) P(H_0) = 0.0078$, $P(+ | H_A) P(H_A) = 0.0298 \Rightarrow h_{MAP} = H_A \equiv \neg \text{Cancer}$





Choosing Hypotheses

- **Bayes's Theorem**

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)} = \frac{P(h \wedge D)}{P(D)}$$

- **MAP Hypothesis**

- * Generally want most probable hypothesis given the training data
- * Define: $\arg \max_{x \in \Omega} [f(x)]$ \equiv value of x in sample space Ω with highest $f(x)$
- * Maximum a posteriori hypothesis, h_{MAP}

$$\begin{aligned} h_{MAP} &= \arg \max_{h \in H} P(h|D) \\ &= \arg \max_{h \in H} \frac{P(D|h)P(h)}{P(D)} \\ &= \arg \max_{h \in H} P(D|h)P(h) \end{aligned}$$

- **ML Hypothesis**

- * Assume that $p(h_i) = p(h_j)$ for all pairs i, j (uniform priors, i.e., $P_H \sim$ Uniform)
- * Can further simplify and choose the maximum likelihood hypothesis, h_{ML}

$$h_{ML} = \arg \max_{h_i \in H} P(D|h_i)$$



Automated Reasoning using Probabilistic Models: Inference Tasks

Simple queries: compute posterior marginal $P(X_i|\mathbf{E}=e)$
e.g., $P(\text{NoGas}|\text{Gauge} = \text{empty}, \text{Lights} = \text{on}, \text{Starts} = \text{false})$

Conjunctive queries: $P(X_i, X_j|\mathbf{E}=e) = P(X_i|\mathbf{E}=e)P(X_j|X_i, \mathbf{E}=e)$

Optimal decisions: decision networks include utility information;
probabilistic inference required for $P(\text{outcome}|\text{action}, \text{evidence})$

Value of information: which evidence to seek next?

Sensitivity analysis: which probability values are most critical?

Explanation: why do I need a new starter motor?

Adapted from slides by S. Russell, UC Berkeley



Graphical Models of Probability

- **Conditional Independence**

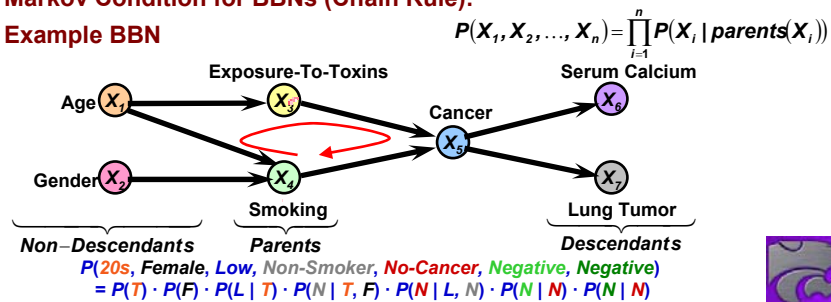
- * X is **conditionally independent (CI)** from Y given Z iff $P(X | Y, Z) = P(X | Z)$ for all values of $X, Y,$ and Z
- * Example: $P(\text{Thunder} | \text{Rain}, \text{Lightning}) = P(\text{Thunder} | \text{Lightning}) \Leftrightarrow T \perp R | L$

- **Bayesian (Belief) Network**

- * **Acyclic directed graph** model $B = (V, E, \Theta)$ representing **CI assertions** over Θ
- * **Vertices** (nodes) V : denote events (each a random variable)
- * **Edges** (arcs, links) E : denote conditional dependencies

- **Markov Condition for BBNs (Chain Rule):**

- **Example BBN**



Semantics of Bayesian Networks

“Global” semantics defines the full joint distribution as the product of the local conditional distributions:

$$P(X_1, \dots, X_n) = \prod_{i=1}^n P(X_i | \text{Parents}(X_i))$$

e.g., $P(J \wedge M \wedge A \wedge \neg B \wedge \neg E)$ is given by??

$$= P(\neg B)P(\neg E)P(A | \neg B \wedge \neg E)P(J | A)P(M | A)$$

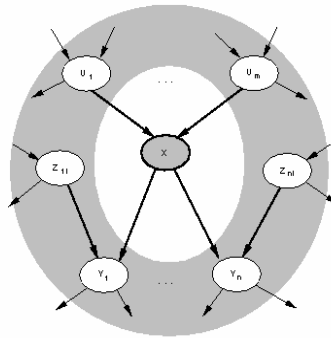
“Local” semantics: each node is conditionally independent of its nondescendants given its parents

Theorem: Local semantics \Leftrightarrow global semantics



Markov Blanket

Each node is conditionally independent of all others given its Markov blanket: parents + children + children's parents



Adapted from slides by S. Russell, UC Berkeley



Constructing Bayesian Networks: The Chain Rule of Inference

Need a method such that a series of locally testable assertions of conditional independence guarantees the required global semantics

1. Choose an ordering of variables X_1, \dots, X_n

2. For $i = 1$ to n

add X_i to the network

select parents from X_1, \dots, X_{i-1} such that

$$\mathbf{P}(X_i | \text{Parents}(X_i)) = \mathbf{P}(X_i | X_1, \dots, X_{i-1})$$

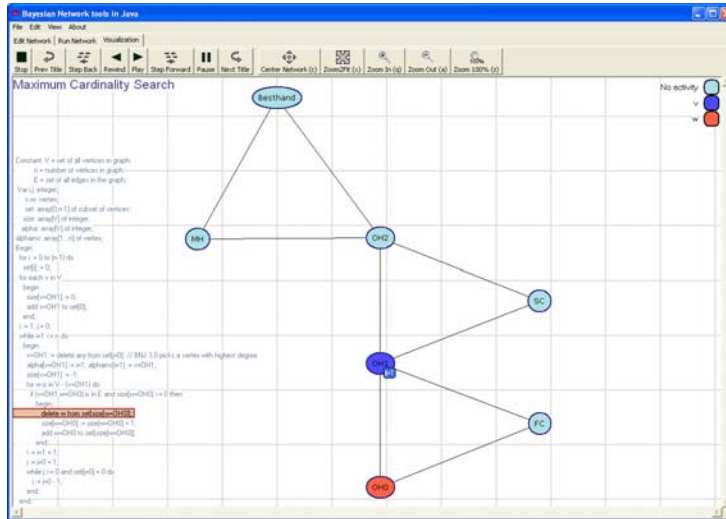
This choice of parents guarantees the global semantics:

$$\begin{aligned} \mathbf{P}(X_1, \dots, X_n) &= \prod_{i=1}^n \mathbf{P}(X_i | X_1, \dots, X_{i-1}) \text{ (chain rule)} \\ &= \prod_{i=1}^n \mathbf{P}(X_i | \text{Parents}(X_i)) \text{ by construction} \end{aligned}$$

Adapted from slides by S. Russell, UC Berkeley



BNJ Visualization [3] Network



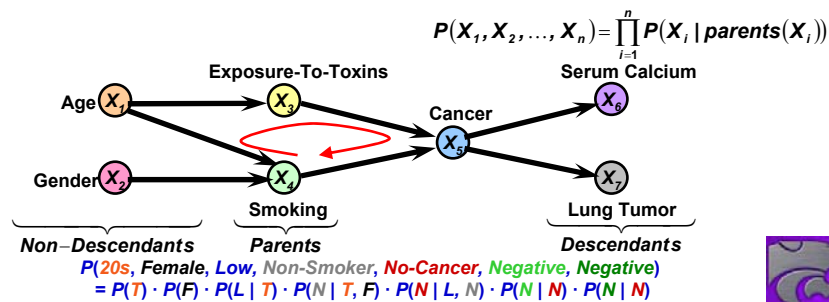
Poker
Network

© 2004 KSU BNJ Development Team



Graphical Models Overview [1]: Bayesian Networks

- **Conditional Independence**
 - * X is **conditionally independent (CI)** from Y given Z (sometimes written $X \perp Y | Z$) if $P(X | Y, Z) = P(X | Z)$ for all values of $X, Y,$ and Z
 - * Example: $P(\text{Thunder} | \text{Rain}, \text{Lightning}) = P(\text{Thunder} | \text{Lightning}) \Leftrightarrow T \perp R | L$
- **Bayesian (Belief) Network**
 - * **Acyclic directed graph** model $B = (V, E, \Theta)$ representing *CI* assertions over Θ
 - * **Vertices** (nodes) V : denote events (each a random variable)
 - * **Edges** (arcs, links) E : denote conditional dependencies
- Markov Condition for BBNs (Chain Rule):
- Example BBN

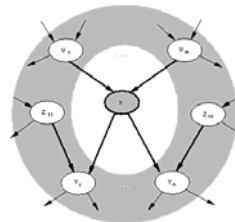




Graphical Models Overview [2]: Markov Blankets and d -Separation Property

Motivation: The conditional independence status of nodes within a BBN might change as the availability of evidence E changes. *Direction-dependent separation (d -separation)* is a technique used to determine conditional independence of nodes as evidence changes.

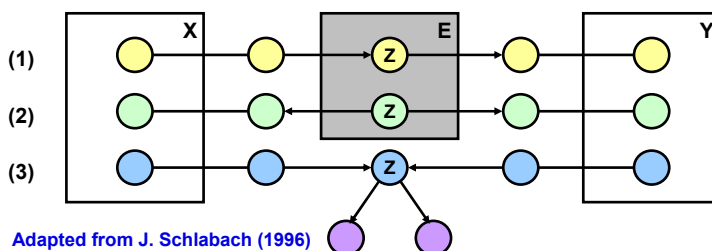
Each node is conditionally independent of all others given its Markov blanket: parents + children + children's parents



Definition: A set of evidence nodes E d -separates two sets of nodes X and Y if every undirected path from a node in X to a node in Y is *blocked* given E .

A path is *blocked* if one of three conditions holds:

From S. Russell & P. Norvig (1995)



Adapted from J. Schlabach (1996)



Graphical Models Overview [3]: Inference Problem

Typically, we are interested in the posterior joint distribution of the query variables \mathbf{Y} given specific values e for the evidence variables \mathbf{E}

Let the hidden variables be $\mathbf{H} = \mathbf{X} - \mathbf{Y} - \mathbf{E}$

Then the required summation of joint entries is done by summing out the hidden variables:

$$P(\mathbf{Y}|\mathbf{E}=e) = \alpha P(\mathbf{Y}, \mathbf{E}=e) = \alpha \sum_{\mathbf{H}} P(\mathbf{Y}, \mathbf{E}=e, \mathbf{H}=\mathbf{h})$$

The terms in the summation are joint entries because \mathbf{Y} , \mathbf{E} , and \mathbf{H} together exhaust the set of random variables

Obvious problems:

- 1) Worst-case time complexity $O(d^n)$ where d is the largest arity
- 2) Space complexity $O(d^n)$ to store the joint distribution
- 3) How to find the numbers for $O(d^n)$ entries???

Multiply-connected case: exact, approximate inference are #P-complete

Adapted from slides by S. Russell, UC Berkeley

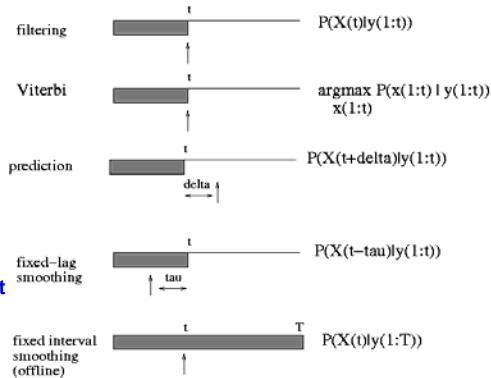
<http://aima.cs.berkeley.edu/>



Other Topics in Graphical Models [1]: Temporal Probabilistic Reasoning

- **Goal: Estimate** $P(X_t^i | y_{1..r})$
- **Filtering: $r = t$**
 - * Intuition: infer current state from observations
 - * Applications: signal identification
 - * Variation: Viterbi algorithm
- **Prediction: $r < t$**
 - * Intuition: infer future state
 - * Applications: prognostics
- **Smoothing: $r > t$**
 - * Intuition: infer past hidden state
 - * Applications: signal enhancement
- **CF Tasks**
 - * Plan recognition by smoothing
 - * Prediction cf. *WebCANVAS* – Cadez et al. (2000)

Adapted from Murphy (2001), Guo (2002)

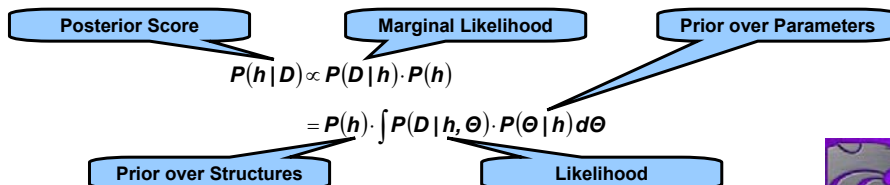


Other Topics in Graphical Models [2]: Learning Structure from Data

- **General-Case BBN Structure Learning: Use Inference to Compute Scores**
- **Optimal Strategy: Bayesian Model Averaging**
 - * Assumption: models $h \in H$ are mutually exclusive and exhaustive
 - * Combine predictions of models in proportion to marginal likelihood
 - Compute conditional probability of hypothesis h given observed data D
 - i.e., compute expectation over unknown h for unseen cases
 - * Let $h \equiv$ structure, parameters $\Theta \equiv$ CPTs

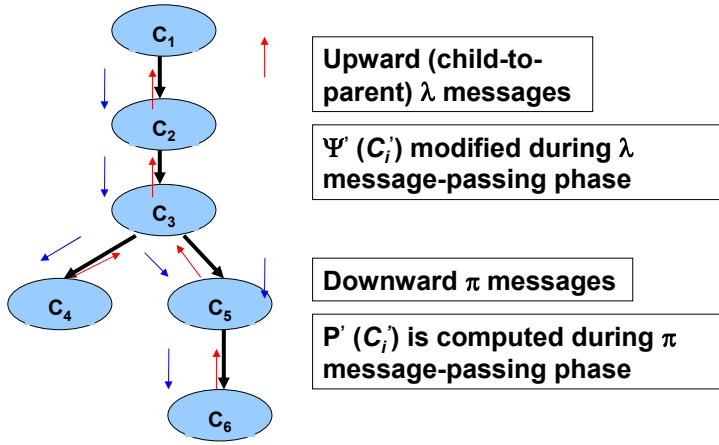
$$P(\bar{x}^{(m+1)} | D) = P(x_1, x_2, \dots, x_n | \bar{x}^{(1)}, \bar{x}^{(2)}, \dots, \bar{x}^{(m)})$$

$$= \sum_{h \in H} P(\bar{x}^{(m+1)} | D, h) \cdot P(h | D)$$





Propagation Algorithm in Singly-Connected Bayesian Networks – Pearl (1983)

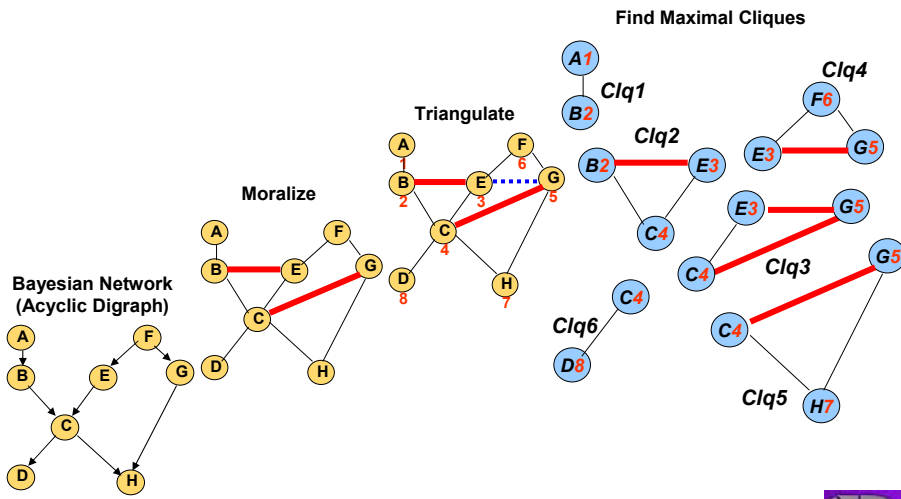


Multiply-connected case: exact, approximate inference are $\#P$ -complete (counting problem is $\#P$ -complete iff decision problem is NP -complete)

Adapted from Neapolitan (1990), Guo (2000)



Inference by Clustering [1]: Graph Operations (Moralization, Triangulation, Maximal Cliques)



Adapted from Neapolitan (1990), Guo (2000)



Inference by Clustering [2]: Function Tree – Lauritzen & Spiegelhalter (1988)

Input: list of cliques of triangulated, moralized graph G_u

Output:

Tree of cliques

Separator nodes S_i ,

Residual nodes R_i and potential probability $\Psi(\text{Clq}_i)$ for all cliques

Algorithm:

1. $S_i = \text{Clq}_i \cap (\text{Clq}_1 \cup \text{Clq}_2 \cup \dots \cup \text{Clq}_{i-1})$
2. $R_i = \text{Clq}_i - S_i$
3. If $i > 1$ then identify a $j < i$ such that Clq_j is a parent of Clq_i
4. Assign each node v to a unique clique Clq_i that $v \cup c(v) \subseteq \text{Clq}_i$
5. Compute $\Psi(\text{Clq}_i) = \prod_{v \in \text{Clq}_i} P(v | c(v))$ {1 if no v is assigned to Clq_i }
6. Store Clq_i , R_i , S_i , and $\Psi(\text{Clq}_i)$ at each vertex in the tree of cliques

Adapted from Neapolitan (1990), Guo (2000)

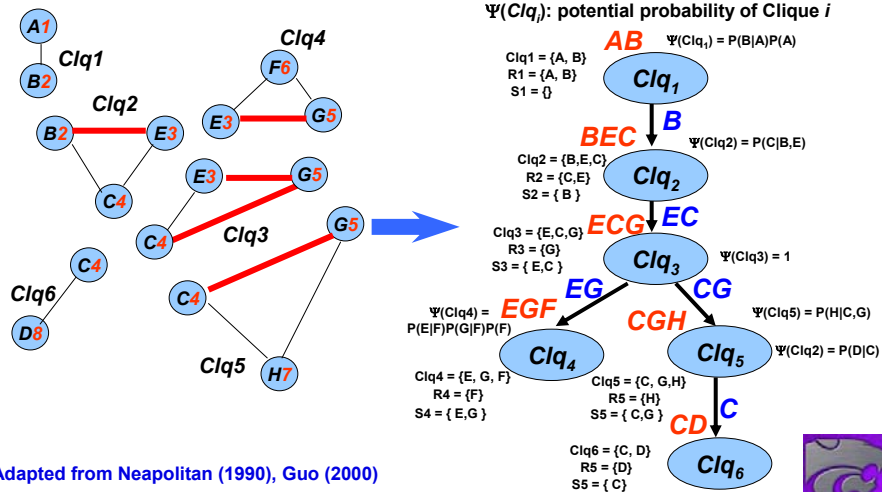


Inference by Clustering [3]: Clique-Tree Operations

R_i : residual nodes

S_i : separator nodes

$\Psi(\text{Clq}_i)$: potential probability of Clique i

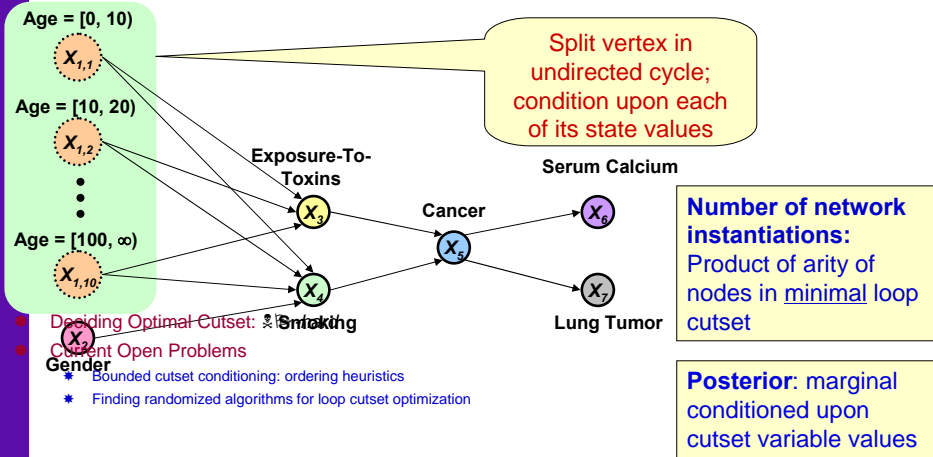


Adapted from Neapolitan (1990), Guo (2000)





Inference by Loop Cutset Conditioning



Inference by Variable Elimination [1]: Intuition

Enumeration is inefficient: repeated computation

e.g., computes $P(J = true|a)P(M = true|a)$ for each value of e

Variable elimination: carry out summations right-to-left, storing intermediate results (factors) to avoid recomputation

$$\begin{aligned}
 P(B|J = true, M = true) &= \alpha \underbrace{P(B)}_B \underbrace{\sum_e P(e)}_E \underbrace{\sum_a P(a|B, e)}_A \underbrace{P(J = true|a)}_J \underbrace{P(M = true|a)}_M \\
 &= \alpha P(B) \sum_e P(e) \sum_a P(a|B, e) P(J = true|a) f_M(a) \\
 &= \alpha P(B) \sum_e P(e) \sum_a P(a|B, e) f_J(a) f_M(a) \\
 &= \alpha P(B) \sum_e P(e) \sum_a f_A(a, b, e) f_J(a) f_M(a) \\
 &= \alpha P(B) \sum_e P(e) f_{\bar{A}JM}(b, e) \text{ (sum out } A) \\
 &= \alpha P(B) f_{\bar{E}\bar{A}JM}(b) \text{ (sum out } E) \\
 &= \alpha f_B(b) \times f_{\bar{E}\bar{A}JM}(b)
 \end{aligned}$$

Adapted from slides by S. Russell, UC Berkeley

<http://aima.cs.berkeley.edu/>



Inference by Variable Elimination [2]: Factoring Operations

Pointwise product of factors f_1 and f_2 :

$$f_1(x_1, \dots, x_j, y_1, \dots, y_k) \times f_2(y_1, \dots, y_k, z_1, \dots, z_l) \\ = f(x_1, \dots, x_j, y_1, \dots, y_k, z_1, \dots, z_l)$$

E.g., $f_1(a, b) \times f_2(b, c) = f(a, b, c)$

Summing out a variable from a product of factors: move any constant factors outside the summation:

$$\sum_x f_1 \times \dots \times f_k = f_1 \times \dots \times f_i \sum_x f_{i+1} \times \dots \times f_k = f_1 \times \dots \times f_i \times f_{\bar{X}}$$

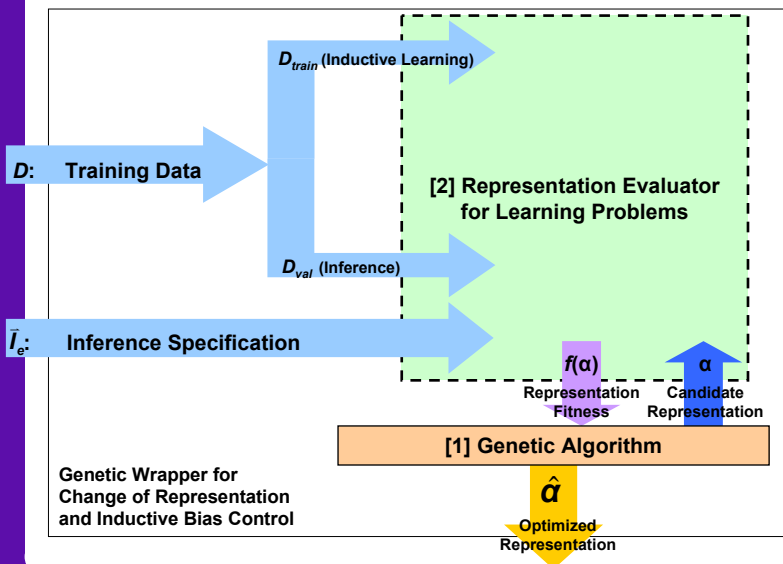
assuming f_1, \dots, f_i do not depend on X

Adapted from slides by S. Russell, UC Berkeley

<http://aima.cs.berkeley.edu/>



Genetic Algorithms for Parameter Tuning in Bayesian Network Structure Learning





Tools for Building Graphical Models

- Commercial Tools: *Ergo*, *Netica*, *TETRAD*, *Hugin*
- **Bayes Net Toolbox (BNT)** – Murphy (1997-present)
 - * Distribution page
<http://http.cs.berkeley.edu/~murphyk/Bayes/bnt.html>
 - * Development group
<http://groups.yahoo.com/group/BayesNetToolbox>
- **Bayesian Network tools in Java (BNJ)** – Hsu et al. (1999-present)
 - * Distribution page <http://bnj.sourceforge.net>
 - * Development group <http://groups.yahoo.com/group/bndev>
 - * Current (re)implementation projects for KSU KDD Lab
 - Continuous state: Minka (2002) – Hsu, Guo, Li
 - Formats: XML BNIF (MSBN), Netica – Barber, Guo
 - Space-efficient DBN inference – Meyer
 - Bounded cutset conditioning – Chandak



References [1]: Graphical Models and Inference Algorithms

- Graphical Models
 - * Bayesian (Belief) Networks tutorial – Murphy (2001)
<http://www.cs.berkeley.edu/~murphyk/Bayes/bayes.html>
 - * Learning Bayesian Networks – Heckerman (1996, 1999)
<http://research.microsoft.com/~heckerman>
- Inference Algorithms
 - * Junction Tree (Join Tree, L-S, *Hugin*): Lauritzen & Spiegelhalter (1988)
<http://citeseer.nj.nec.com/huang94inference.html>
 - * (Bounded) Loop Cutset Conditioning: Horvitz & Cooper (1989)
<http://citeseer.nj.nec.com/shachter94global.html>
 - * Variable Elimination (Bucket Elimination, *ElimBel*): Dechter (1986)
<http://citeseer.nj.nec.com/dechter96bucket.html>
 - * Recommended Books
 - Neapolitan (1990) – *out of print*; see Pearl (1988), Jensen (2001)
 - Castillo, Gutierrez, Hadi (1997)
 - Cowell, Dawid, Lauritzen, Spiegelhalter (1999)
 - * Stochastic Approximation
<http://citeseer.nj.nec.com/cheng00aisbn.html>



References [2]: Machine Learning, KDD, and Bioinformatics

- **Machine Learning, Data Mining, and Knowledge Discovery**
 - * **K-State KDD Lab: literature survey and resource catalog (1999-present)**
<http://www.kddresearch.org/Resources>
 - * **Bayesian Network tools in Java (BNJ): Hsu, Barber, King, Meyer, Thornton (2002-present)**
<http://bnj.sourceforge.net>
 - * **Machine Learning in Java (BNJ): Hsu, Louis, Plummer (2002)**
<http://mldev.sourceforge.net>
- **Bioinformatics**
 - * **European Bioinformatics Institute Tutorial: Brazma et al. (2001)**
http://www.ebi.ac.uk/microarray/biology_intro.htm
 - * **Hebrew University: Friedman, Pe'er, et al. (1999, 2000, 2002)**
<http://www.cs.huji.ac.il/labs/compbio/>
 - * **K-State BMI Group: literature survey and resource catalog (2002-2005)**
<http://www.kddresearch.org/Groups/Bioinformatics>



Terminology

- **Introduction to Reasoning under Uncertainty**
 - * **Probability foundations**
 - * **Definitions: subjectivist, frequentist, logicist**
 - * **(3) Kolmogorov axioms**
- **Bayes's Theorem**
 - * **Prior probability of an event**
 - * **Joint probability of an event**
 - * **Conditional (posterior) probability of an event**
- **Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Hypotheses**
 - * **MAP hypothesis: highest conditional probability given observations (data)**
 - * **ML: highest likelihood of generating the observed data**
 - * **ML estimation (MLE): estimating parameters to find ML hypothesis**
- **Bayesian Inference: Computing Conditional Probabilities (CPs) in A Model**
- **Bayesian Learning: Searching Model (Hypothesis) Space using CPs**





Summary Points

- **Introduction to Probabilistic Reasoning**
 - * **Framework:** using probabilistic criteria to search H
 - * **Probability foundations**
 - ⇒ Definitions: subjectivist, objectivist; Bayesian, frequentist, logicist
 - ⇒ Kolmogorov axioms
- **Bayes's Theorem**
 - * Definition of conditional (posterior) probability
 - * Product rule
- **Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Hypotheses**
 - * Bayes's Rule and MAP
 - * Uniform priors: allow use of MLE to generate MAP hypotheses
 - * Relation to version spaces, candidate elimination
- **Next Week: Chapter 14, Russell and Norvig**
 - * Later: Bayesian learning: MDL, BOC, Gibbs, Simple (Naïve) Bayes
 - * Categorizing text and documents, other applications

