Lecture 10 of 42

Logical Agents and Propositional Logic
Discussion: Logic in AI

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KSOL course page: http://snipurl.com/v9v3
Course web site: http://www.kddresearch.org/Courses/Fall-2007/CIS730
Instructor home page: http://www.cis.ksu.edu/~bhsu

Reading for Next Class:
Section 7.5 – 7.7, p. 211 - 232, Russell & Norvig 2nd edition

Lecture Outline

- Reading for Next Class: Sections 7.5 – 7.7, R&N 2e
- Today: Logical Agents
  - Classical knowledge representation
  - Limitations of the classical symbolic approach
  - Modern approach: representation, reasoning, learning
  - “New” aspects: uncertainty, abstraction, classification paradigm
- Next Week: Start of Material on Logic
  - Representation: “a bridge between learning and reasoning” (Koller)
  - Basis for automated reasoning: theorem proving, other inference

Overview

- **Today's Reading**
  - **Sections 7.1 – 7.4, Russell and Norvig 2e**
  - **Recommended references: Nilsson and Genesereth (*Logical Foundations of AI*)**
- **Previously: Logical Agents**
  - **Knowledge Bases (KB) and KB agents**
  - **Motivating example: Wumpus World**
  - **Logic in general**
  - **Syntax of propositional calculus**
- **Today**
  - **Propositional calculus (concluded)**
  - **Normal forms**
  - **Production systems**
  - **Predicate logic**
  - **Introduction to First-Order Logic (FOL): examples, inference rules (sketch)**
- **Next Week: First-Order Logic Review, Resolution**

Knowledge Representation (KR) for Intelligent Agent Problems

- Percepts
  - *What can agent observe?*
  - *What can sensors tell it?*
- Actions
  - *What actuators does agent have?*
  - *In what context are they applicable?*
- Goals
  - *What are agents goals? Preferences (utilities)?*
  - *How does agent evaluate them (check environment, deliberate, etc.)?*
- Environment
  - *What are "rules of the world"?*
  - *How can these be represented, simulated?*

Review: Simple Knowledge-Based Agent

```
function KB-AGENT( percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE( action, t))
   t ← t + 1
   return action
```

The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Adapted from slides by S. Russell, UC Berkeley

Figure 6.1 p. 152 R&N

Review: Types of Logic

Logics are characterized by what they commit to as "primitives"

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

Adapted from slides by S. Russell, UC Berkeley

Figure 6.7 p. 166 R&N

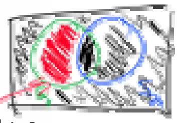
Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. A B C
$True$ $True$ $False$

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true and	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true or	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false or	S_2	is true
i.e.	is false iff	S_1	is true and	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true and	$S_2 \Rightarrow S_1$	is true



Adapted from slides by S. Russell, UC Berkeley

Propositional Inference: Enumeration (Model Checking) Method

Let $\alpha = A \vee B$ and $KB = (A \vee C) \wedge (B \vee \neg C)$

Is it the case that $KB \models \alpha$?
Check all possible models—α must be true wherever KB is true

A	B	C	$A \vee C$	$B \vee \neg C$	KB	α
$False$	$False$	$False$	$False$	$True$	$False$	$False$
$False$	$False$	$True$	$True$	$False$	$False$	$False$
$False$	$True$	$False$	$False$	$True$	$False$	$True$
$False$	$True$	$True$	$True$	$True$	$True$	$True$
$True$	$False$	$False$	$True$	$True$	$True$	$True$
$True$	$False$	$True$	$True$	$False$	$False$	$True$
$True$	$True$	$False$	$True$	$True$	$True$	$True$
$True$	$True$	$True$	$True$	$True$	$True$	$True$

Adapted from slides by S. Russell, UC Berkeley

Normal Forms: CNF, DNF, Horn

Other approaches to inference use syntactic operations on sentences, often expressed in standardized forms

Conjunctive Normal Form (CNF—universal)

conjunction of *disjunctions* of *literals* (*clauses*)

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Disjunctive Normal Form (DNF—universal)

disjunction of *conjunctions* of *literals* (*terms*)

E.g., $(A \wedge B) \vee (A \wedge \neg C) \vee (A \wedge \neg D) \vee (\neg B \wedge \neg C) \vee (\neg B \wedge \neg D)$

Horn Form (restricted)

conjunction of *Horn clauses* (clauses with ≤ 1 positive literal)

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Often written as set of implications:

$B \Rightarrow A$ and $(C \wedge D) \Rightarrow B$

Adapted from slides by S. Russell, UC Berkeley

Validity and Satisfiability

A sentence is valid if it is true in all models

e.g., $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

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Proof Methods

Proof methods divide into (roughly) two kinds:

Model checking

- truth table enumeration (sound and complete for propositional)
- heuristic search in model space (sound but incomplete)
  - e.g., the GSAT algorithm (Ex. 6.15)

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
  - Can use inference rules as operators in a standard search alg.

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Inference (Sequent) Rules for Propositional Logic

Resolution (for CNF): complete for propositional logic

$$\frac{\alpha \vee \beta, \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining

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Logical Agents: Taking Stock

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- $\models$ entailment: necessary truth of one sentence given another
- $\vdash$ inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks

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The Road Ahead: Predicate Logic and FOL

- Predicate Logic
  - Enriching language
    - Predicates
    - Functions
  - Syntax and semantics of predicate logic
- First-Order Logic (FOL, FOPC)
  - Need for quantifiers
  - Relation to (unquantified) predicate logic
  - Syntax and semantics of FOL
- Fun with Sentences
- Wumpus World in FOL

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Syntax of FOL: Basic Elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge\ \vee\ \neg\ \Rightarrow\ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

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FOL: Atomic Sentences (Atomic Well-Formed Formulae)

Atomic sentence $=$ $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $=$ $function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)$

Adapted from slides by S. Russell, UC Berkeley

Summary Points

- Logical Agents Overview (Last Time)
  - Knowledge Bases (KB) and KB agents
  - Motivating example: Wumpus World
  - Logic in general
  - Syntax of propositional calculus
- Propositional and First-Order Calculi (Today)
  - Propositional calculus (concluded)
    - Normal forms
    - Inference (*aka* sequent) rules
  - Production systems
  - Predicate logic without quantifiers
  - Introduction to First-Order Logic (FOL)
    - Examples
    - Inference rules (sketch)
- Next Week: First-Order Logic Review, Intro to Resolution Theorem Proving

Fun with Sentences: Family Feud

- **Brothers are Siblings**
  - $\forall x, y \,.\, \mathit{Brother}(x, y) \Leftrightarrow \mathit{Sibling}(x, y)$
- **Siblings (i.e., Sibling Relationships) are Reflexive**
  - $\forall x, y \,.\, \mathit{Sibling}(x, y) \Leftrightarrow \mathit{Sibling}(y, x)$
- **One's Mother is One's Female Parent**
  - $\forall x, y \,.\, \mathit{Mother}(x, y) \Leftrightarrow \mathit{Female}(x) \land \mathit{Parent}(x, y)$
- **A *First Cousin* Is A Child of A Parent's Sibling**
  - $\forall x, y \,.\, \mathit{First\text{-}Cousin}(x, y) \Leftrightarrow$
    $\exists p, ps \,.\, \mathit{Parent}(p, x) \land \mathit{Sibling}(p, ps) \land \mathit{Parent}(ps, y)$

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Jigsaw Exercise [1]: First-Order Logic Sentences

- "Every Dog Chases Its Own Tail"
  - ∀ *d* . Chases (*d*, *tail-of* (d))
  - Alternative Statement: ∀ *d* . ∃ *t* . *Tail-Of* (*t*, *d*) ∧ *Chases* (*d*, *t*)
  - Prefigures concept of Skolemization (Skolem variables / functions)
- "Every Dog Chases Its Own (Unique) Tail"
  - ∀ *d* . $\exists^1$ *t* . Tail-Of (*t*, *d*) ∧ Chases (*d*, *t*) ≡ ∀ *d* . ∃ *t* . Tail-Of (*t*, *d*) ∧ Chases (*d*, *t*) ∧ [∀ *t'* Chases (*d*, *t'*) ⇒ *t'* = *t*]
- "Only The Wicked Flee when No One Pursueth"
  - ∀ *x* . *Flees* (*x*) ∧ [¬∃ *y* *Pursues* (*y*, *x*)] ⇒ *Wicked* (*x*)
  - Alternative : ∀ *x* . [∃ *y* . *Flees* (*x*, *y*)] ∧ [¬∃ *z* . *Pursues* (*z*, *x*)] ⇒ *Wicked* (*x*)
- Offline Exercise: What Is An *n*th Cousin, *m* Times Removed?

Jigsaw Exercise [2]: First-Order Logic Sentences

Terminology

- Logical Frameworks
  - Knowledge Bases (KB)
  - Logic in general: representation languages, syntax, semantics
  - Propositional logic
  - First-order logic (FOL, FOPC)
  - Model theory, domain theory: possible worlds semantics, entailment
- Normal Forms
  - Conjunctive Normal Form (CNF)
  - Disjunctive Normal Form (DNF)
  - Horn Form
- Proof Theory and Inference Systems
  - Sequent calculi: rules of proof theory
  - Derivability or provability
  - Properties
    - Soundness (derivability implies entailment)
    - Completeness (entailment implies derivability)