Lecture 11 of 42

Resolution, Forward/Backward Chaining
Discussion: Expert Systems

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KSOL course page: http://snipurl.com/v9v3

Course web site: http://www.kddresearch.org/Courses/Fall-2007/CIS730

Instructor home page: http://www.cis.ksu.edu/~bhsu

Reading for Next Class:

Section 7.5 – 7.7, p. 211 - 232, Russell & Norvig 2nd edition

Normal Forms: CNF, DNF, Horn

Other approaches to inference use syntactic operations on sentences, often expressed in standardized forms

Conjunctive Normal Form (CNF—universal)

conjunction of *disjunctions of literals* (clauses)

E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

Disjunctive Normal Form (DNF—universal)

disjunction of *conjunctions of literals* (terms)

E.g., $(A \land B) \lor (A \land \neg C) \lor (A \land \neg D) \lor (\neg B \land \neg C) \lor (\neg B \land \neg D)$

Horn Form (restricted)

conjunction of Horn clauses (clauses with ≤ 1 positive literal)

E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

Often written as set of implications:

$B \Rightarrow A$ and $(C \land D) \Rightarrow B$

KB ⊨ S entailment

KB ⊢ S provability / derivability

Properties of KB + semantics

Soundness

Completeness

Proof proc.

Satisfiability

Validity

Sentences (KB) + semantics

antecedent consequent

$B \Rightarrow A$

$A :- B$

NP-complete

Adapted from slides by S. Russell, UC Berkeley

Validity and Satisfiability

Ex Falso Quodlibet

Q.E.D. quod erat demonstrandum

A sentence is valid if it is true in all models

e.g., $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

$\models \alpha \Leftrightarrow \alpha$ is valid (empty KB)

A sentence is satisfiable if it is true in some model

e.g., $A \lor B$, C

$\exists w \in [\![A]\!]$

A sentence is unsatisfiable if it is true in no models

e.g., $A \land \neg A$

$\neg \exists w \in [\![A]\!] \Leftrightarrow \forall w .\ w \notin [\![A]\!]$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg \alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

$\boxed{A \lor \neg A}$ Law of the Excluded Middle (true in Classical Logic)

Suppose For The Sake of Contradiction

Without Loss Of Generality

Adapted from slides by S. Russell, UC Berkeley

Proof Methods

Proof methods divide into (roughly) two kinds:

$$\frac{p \rightarrow q \quad p}{\therefore q}$$

Model checking

- truth table enumeration (sound and complete for propositional)
- heuristic search in model space (sound but incomplete)
  e.g., the GSAT algorithm (Ex. 6.15)

Application of inference rules

GMP

Legitimate (sound) generation of new sentences from old

Proof = a sequence of inference rule applications

Can use inference rules as operators in a standard search alg.

Adapted from slides by S. Russell, UC Berkeley

Inference (Sequent) Rules for Propositional Logic

$$\frac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$$

$\beta \vee \beta$

$\neg(\neg\beta) \vee \beta$ $\quad \neg A \Rightarrow \beta$

Resolution (for CNF): complete for propositional logic

$$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

$$\frac{\neg A \Rightarrow B \quad B \Rightarrow C}{\neg A \Rightarrow C}$$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \quad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining

Adapted from slides by S. Russell, UC Berkeley

Logical Agents: Taking Stock

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

KB

Proof rules

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks

Adapted from slides by S. Russell, UC Berkeley

The Road Ahead: Predicate Logic and FOL

- **Predicate Logic**
  - **Enriching language**
    - **Predicates**
    - **Functions**
  - **Syntax and semantics of predicate logic**
- **First-Order Logic (FOL, FOPC)**
  - **Need for quantifiers**
  - **Relation to (unquantified) predicate logic**
  - **Syntax and semantics of FOL**
- **Fun with Sentences**
- **Wumpus World in FOL**

$P(x_1, x_2, \ldots, x_j)$.

TimIsAwake | Propos.

Awake(Tim). | Pred.

Adapted from slides by S. Russell, UC Berkeley

Syntax of FOL: Basic Elements

Constants	$KingJohn$, 2, UCB, ...
Predicates	$Brother$, >, ...
Functions	$Sqrt$, $LeftLegOf$, ...
Variables	x, y, a, b, ...
Connectives	$\wedge \; \vee \; \neg \; \Rightarrow \; \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall \; \exists$

Objects

Relation = a set of tuples for which the pred. holds

$[\![f]\!] = \{(x,y) \mid f(x)=y\}$

$\forall x \; P(x)$

$\Leftrightarrow P(x_1) \wedge P(x_2) \wedge \ldots$

$\exists x \; P(x)$

$\Leftrightarrow P(x_1) \vee P(x_2) \vee \ldots$

Adapted from slides by S. Russell, UC Berkeley

FOL: Atomic Sentences (Atomic Well-Formed Formulae)

Atomic sentence = $predicate(term_1, \ldots, term_n)$ or $term_1 = term_2$ 0-ary

Term = $function(term_1, \ldots, term_n)$ or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn))$

pred func func

term

term

Atom

Adapted from slides by S. Russell, UC Berkeley

Summary Points

- Logical Agents Overview (Last Time)
  - Knowledge Bases (KB) and KB agents
  - Motivating example: Wumpus World
  - Logic in general
  - Syntax of propositional calculus
- Propositional and First-Order Calculi (Today)
  - Propositional calculus (concluded)
    - Normal forms
    - Inference (*aka* sequent) rules
  - Production systems
  - Predicate logic without quantifiers
  - Introduction to First-Order Logic (FOL)
    - Examples
    - Inference rules (sketch)
- Next Week: FOL Review, Intro to Resolution

Fun with Sentences: Family Feud

- **Brothers <u>are</u> Siblings**
  - $\forall x, y \,.\, Brother(x, y) \Leftrightarrow Sibling(x, y)$
- **Siblings (i.e., Sibling Relationships) are Reflexive**
  - $\forall x, y \,.\, Sibling(x, y) \Leftrightarrow Sibling(y, x)$
- **One's Mother is One's Female Parent**
  - $\forall x, y \,.\, Mother(x, y) \Leftrightarrow Female(x) \wedge Parent(x, y)$
- **A *First Cousin* Is A Child of A Parent's Sibling**
  - $\forall x, y \,.\, First\text{-}Cousin(x, y) \Leftrightarrow$
    $\exists p, ps \,.\, Parent(p, x) \wedge Sibling(p, ps) \wedge Parent(ps, y)$

Adapted from slides by S. Russell, UC Berkeley

Jigsaw Exercise [1]: First-Order Logic Sentences

- **"Every Dog Chases Its Own Tail"**
  - $\forall d \,.\, Chases(d, tail\text{-}of(d))$
  - Alternative Statement: $\forall d \,.\, \exists t \,.\, Tail\text{-}Of(t, d) \wedge Chases(d, t)$
  - Prefigures concept of <u>Skolemization</u> (Skolem vars / functions)
- **"Every Dog Chases Its Own (Unique) Tail"**
  - $\forall d \,.\, \exists^1 t \,.\, Tail\text{-}Of(t, d) \wedge Chases(d, t) \equiv$
    $\forall d \,.\, \exists t \,.\, Tail\text{-}Of(t, d) \wedge Chases(d, t) \wedge [\forall t'\, Chases(d, t') \Rightarrow t' = t]$
- **"Only The Wicked Flee when No One Pursueth"**
  - $\forall x \,.\, Flees(x) \wedge [\neg\exists y\, Pursues(y, x)] \Rightarrow Wicked(x)$
  - Alternative : $\forall x \,.\, [\exists y \,.\, Flees(x, y)] \wedge [\neg\exists z \,.\, Pursues(z, x)] \Rightarrow Wicked(x)$
- **Offline Exercise: What Is An *n*th Cousin, *m* Times Removed?**

Jigsaw Exercise [2]: First-Order Logic Sentences

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- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
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FOL: Complex Sentences (Well-Formed Formulae)

Complex sentences are made from atomic sentences using connectives

$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg>(1,2)$

Adapted from slides by S. Russell, UC Berkeley

Truth in FOL

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for
 constant symbols → objects
 predicate symbols → relations
 function symbols → functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Adapted from slides by S. Russell, UC Berkeley

Terminology

- Logical Frameworks
  - Knowledge Bases (KB)
  - Logic in general: representation languages, syntax, semantics
  - Propositional logic
  - First-order logic (FOL, FOPC)
  - Model theory, domain theory: possible worlds semantics, entailment
- Normal Forms
  - Conjunctive Normal Form (CNF)
  - Disjunctive Normal Form (DNF)
  - Horn Form
- Proof Theory and Inference Systems
  - Sequent calculi: rules of proof theory
  - Derivability or provability
  - Properties
    - Soundness (derivability implies entailment)
    - Completeness (entailment implies derivability)