Lecture 13 of 42

First-Order Logic: KE and Theorem Proving
Discussion: Review of Models, Theorem Proving

Friday, 21 September 2007

William H. Hsu

Department of Computing and Information Sciences, KSU

KSOL course page: http://snipurl.com/v9v3

Course web site: http://www.kddresearch.org/Courses/Fall-2007/CIS730

Instructor home page: http://www.cis.ksu.edu/~bhsu

Reading for Next Class:

Section 8.3 – 8.5, p. 240 – 268, Russell & Norvig 2nd edition

Section 9.1, p. 272 – 275, Russell & Norvig 2nd edition

Lecture Outline

- Reading for Next Class: Section 8.1 – 8.2, R&N 2e
- Recommended : Nilsson and Genesereth (Chapter 5 online)
- Next Week's: Chapter 8 & first half of Chapter 9, R&N
- Today: Knowledge Engineering and Theorem Proving
- Next Week
  - Resolution
  - Constraint logic
  - Prolog
- Week of ~~04~~ Oct ~~2006~~
  - Knowledge representation
  - Ontologies

Logical Agents: Review

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

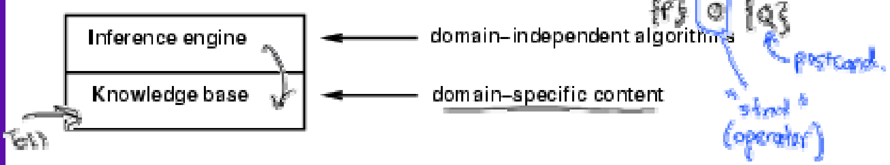
Propositional logic suffices for some of these tasks

Adapted from slides by
S. Russell, UC Berkeley

Predicate Logic and FOL Road Map

- Predicate Logic
  - Enriching language
    - Predicates
    - Functions
  - Syntax and semantics of predicate logic
- First-Order Logic (FOL, FOPC)
  - Need for quantifiers
  - Relation to (unquantified) predicate logic
  - Syntax and semantics of FOL
- Fun with Sentences
- Wumpus World in FOL

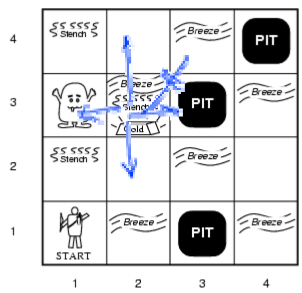
Knowledge bases: Review



- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - Tell it what it needs to know
- Then it can Ask itself what to do - answers should follow from KB
- Agents can be viewed at the knowledge level
  i.e., what they know, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

Wumpus World – PEAS Description: Review

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Wumpus world – characterization: Review

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

Logic in General: Review

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

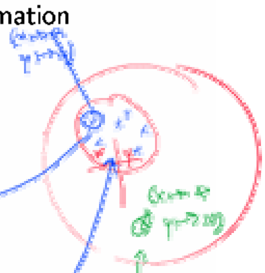
Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$



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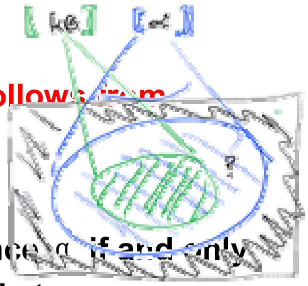
Entailment: Review

- **Entailment** means that one thing **follows from** another:

KB ⊨ α



- **Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true**
  - E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"
  - E.g., x+y = 4 entails 4 = x+y
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

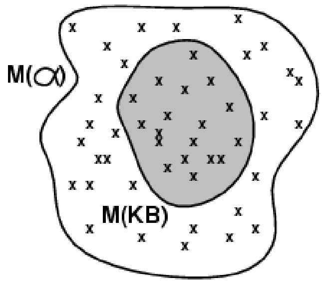
DPLL algorithm: Review

```
function DPLL-SATISFIABLE?(s) returns true or false
   inputs: s, a sentence in propositional logic

   clauses ← the set of clauses in the CNF representation of s
   symbols ← a list of the proposition symbols in s
   return DPLL(clauses, symbols, [])

function DPLL(clauses, symbols, model) returns true or false

   if every clause in clauses is true in model then return true
   if some clause in clauses is false in model then return false
   P, value ← FIND-PURE-SYMBOL(symbols, clauses, model)
   if P is non-null then return DPLL(clauses, symbols–P, [P = value|model])
   P, value ← FIND-UNIT-CLAUSE(clauses, model)
   if P is non-null then return DPLL(clauses, symbols–P, [P = value|model])
   P ← FIRST(symbols); rest ← REST(symbols)
   return DPLL(clauses, rest, [P = true|model]) or
          DPLL(clauses, rest, [P = false|model])
```

Figure 7.16 p. 222 R&N 2e

Models: Review

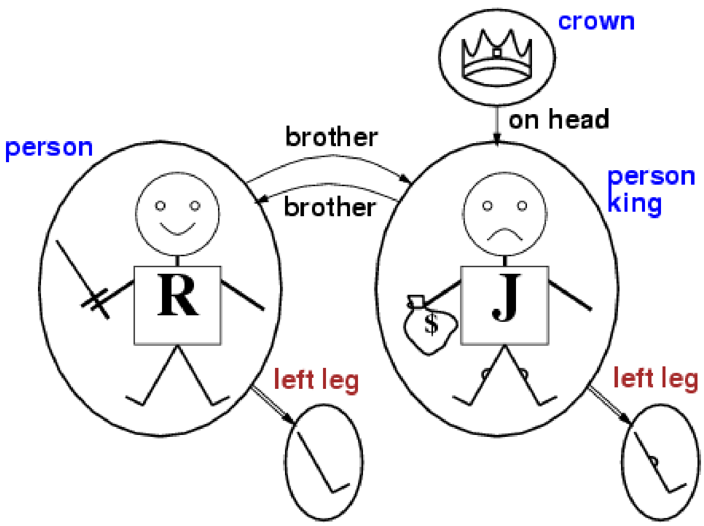
- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB $\models$ α iff $M(KB) \subseteq M(\alpha)$
  - E.g. *KB* = Giants won and Reds won α = Giants won
- See: definitions on p. 201, 203 R&N 2e



Adapted from slide

See also: S. 7.3, p. 200 – 204
S. 8.2, p. 245 – 253

Models for FOL: Example



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Types of Logic: Review

Logics are characterized by what they commit to as "primitives"

Ontological commitment: what exists—facts? objects? time? beliefs?

Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

Adapted from slides by S. Russell, UC Berkeley

Figure 8.1 p. 244 R&N 2e

FOL – Atomic Sentences (Atoms): Review

Atomic sentence = $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term = $function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn))$

Adapted from slides by S. Russell, UC Berkeley

FOL – Complex Sentences (WFFs): Review

Complex sentences are made from atomic sentences using connectives

$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg>(1,2)$

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S. Russell, UC Berkeley

Truth in FOL: Review

Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for
constant symbols → objects
predicate symbols → relations
function symbols → functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

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Automated Deduction (Chapters 8-10): Review

Sound inference: find α such that $KB \models \alpha$.
Proof process is a search, operators are inference rules.

E.g., Modus Ponens (MP)

$$\frac{\alpha, \quad \alpha \Rightarrow \beta}{\beta} \qquad \frac{At(Joe, UCB) \quad At(Joe, UCB) \Rightarrow OK(Joe)}{OK(Joe)}$$

E.g., And-Introduction (AI)

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{OK(Joe) \quad CSMajor(Joe)}{OK(Joe) \wedge CSMajor(Joe)}$$

E.g., Universal Elimination (UE)

$$\frac{\forall x \;\; \alpha}{\alpha\{x/\tau\}} \qquad \frac{\forall x \;\; At(x, UCB) \Rightarrow OK(x)}{At(Pat, UCB) \Rightarrow OK(Pat)}$$

τ must be a ground term (i.e., no variables)

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Example Proof

Bob is a buffalo	1. $Buffalo(Bob)$
Pat is a pig	2. $Pig(Pat)$
Buffaloes outrun pigs	3. $\forall x, y \;\; Buffalo(x) \wedge Pig(y) \Rightarrow Faster(x, y)$
Bob outruns Pat	

- Apply Sequent Rules to Generate New Assertions

AI 1 & 2	4. $Buffalo(Bob) \wedge Pig(Pat)$
UE 3, $\{x/Bob, y/Pat\}$	5. $Buffalo(Bob) \wedge Pig(Pat) \Rightarrow Faster(Bob, Pat)$
MP 6 & 7	6. $Faster(Bob, Pat)$

$$\frac{\alpha, \quad \alpha \Rightarrow \beta}{\beta} \qquad \frac{\alpha \quad \beta}{\alpha \wedge \beta} \qquad \frac{\forall x \;\; \alpha}{\alpha\{x/\tau\}}$$

- Modus Ponens — And Introduction — Universal Elimination

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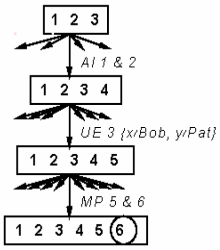
Search with Primitive Inference Rules

Operators are inference rules
States are sets of sentences
Goal test checks state to see if it contains query sentence



AI, UE, MP is a common inference pattern

Problem: branching factor huge, esp. for UE

Idea: find a substitution that makes the rule premise match some known facts
⇒ a single, more powerful inference rule

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A Brief History of Reasoning: Chapter 8 End Notes, R&N

450B.C.	Stoics	propositional logic, inference (maybe)
322B.C.	Aristotle	"syllogisms" (inference rules), quantifiers
1565	Cardano	probability theory (propositional logic + uncertainty)
1847	Boole	propositional logic (again)
1879	Frege	first-order logic
1922	Wittgenstein	proof by truth tables
1930	Gödel	∃ complete algorithm for FOL
1930	Herbrand	complete algorithm for FOL (reduce to propositional)
1931	Gödel	¬∃ complete algorithm for arithmetic
1960	Davis/Putnam	"practical" algorithm for propositional logic
1965	Robinson	"practical" algorithm for FOL—resolution

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Knowledge Engineering

- **KE: Process of**
  - Choosing logical language (basis of KR)
  - Building KB
  - Implementing proof theory
  - Inferring new facts
- **Analogy: Programming Languages / Software Engineering**
  - Choosing programming language (basis of software engineering)
  - Writing program
  - Choosing / writing compiler
  - Running program
- **Example Domains**
  - Electronic circuits (Section 8.3 R&N)
  - Exercise
    - ⇨ Look up, read about <u>protocol analysis</u>
    - ⇨ Find example and think about KE process for your project domain

Unification: Definitions and Idea Sketch

A substitution σ unifies atomic sentences p and q if $p\sigma = q\sigma$

p	q	σ
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/John, y/OJ\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$

<u>Idea</u>: Unify rule premises with known facts, apply unifier to conclusion

E.g., if we know q and $Knows(John, x) \Rightarrow Likes(John, x)$

then we conclude $Likes(John, Jane)$

$Likes(John, OJ)$

$Likes(John, Mother(John))$

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Generalized Modus Ponens

$$\frac{p_1{}',\ p_2{}',\ \ldots,\ p_n{}',\ (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\sigma} \qquad \text{where } p_i{}'\sigma = p_i\sigma \text{ for all } i$$

E.g. $p_1{}' =$ Faster(Bob,Pat)

$p_2{}' =$ Faster(Pat,Steve)

$p_1 \wedge p_2 \Rightarrow q = Faster(x,y) \wedge Faster(y,z) \Rightarrow Faster(x,z)$

$\sigma = \{x/Bob, y/Pat, z/Steve\}$

$q\sigma = Faster(Bob, Steve)$

GMP used with KB of definite clauses (*exactly* one positive literal):
either a single atomic sentence or
(conjunction of atomic sentences) $\Rightarrow$ (atomic sentence)
All variables assumed universally quantified

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Soundness of GMP

Need to show that

$$p_1{}',\ \ldots,\ p_n{}',\ (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\sigma$$

provided that $p_i{}'\sigma = p_i\sigma$ for all i

Lemma: For any definite clause p, we have $p \models p\sigma$ by UE

1. $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\sigma = (p_1\sigma \wedge \ldots \wedge p_n\sigma \Rightarrow q\sigma)$
2. $p_1{}',\ \ldots,\ p_n{}' \models p_1{}' \wedge \ldots \wedge p_n{}' \models p_1{}'\sigma \wedge \ldots \wedge p_n{}'\sigma$
3. From 1 and 2, $q\sigma$ follows by simple MP

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Forward Chaining

When a new fact p is added to the KB
  for each rule such that p unifies with a premise
    if the other premises are <u>known</u>
    then add the conclusion to the KB and continue chaining

Forward chaining is <u>data-driven</u>
    e.g., inferring properties and categories from percepts

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S. Russell, UC Berkeley

Example: Forward Chaining

Add facts 1, 2, 3, 4, 5, 7 in turn.
Number in [] = unification literal; $\surd$ indicates rule firing

1. $Buffalo(x) \wedge Pig(y) \Rightarrow Faster(x, y)$
2. $Pig(y) \wedge Slug(z) \Rightarrow Faster(y, z)$
3. $Faster(x, y) \wedge Faster(y, z) \Rightarrow Faster(x, z)$
4. $Buffalo(Bob)$ [1a, ×]
5. $Pig(Pat)$ [1b, $\surd$] → 6. $Faster(Bob, Pat)$ [3a, ×], [3b, ×]
   [2a, ×]
7. $Slug(Steve)$ [2b, $\surd$]
   → 8. $Faster(Pat, Steve)$ [3a, ×], [3b, $\surd$]
     → 9. $Faster(Bob, Steve)$ [3a, ×], [3b, ×]

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Backward Chaining

When a query q is asked
- if a matching fact q' is known, return the unifier
- for each rule whose consequent q' matches q
  - attempt to prove each premise of the rule by backward chaining

(Some added complications in keeping track of the unifiers)

(More complications help to avoid infinite loops)

Two versions: find any solution, find all solutions

Backward chaining is the basis for logic programming, e.g., Prolog

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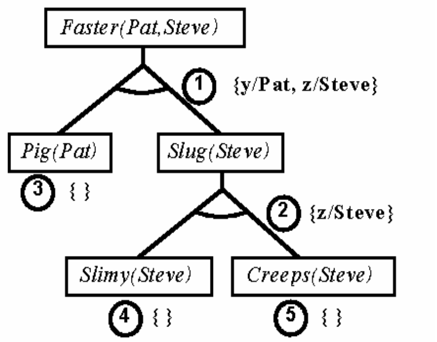
Example: Backward Chaining

1. $Pig(y) \land Slug(z) \Rightarrow Faster(y, z)$
2. $Slimy(z) \land Creeps(z) \Rightarrow Slug(z)$
3. $Pig(Pat)$ 4. $Slimy(Steve)$ 5. $Creeps(Steve)$



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Two versions: find <u>any</u> solution, find <u>all</u> solutions

Backward chaining is the basis for <u>logic programming</u>, e.g., Prolog

- <u>Answer</u>
  - Suppose ¬*Query*, <u>F</u>or <u>T</u>he <u>S</u>ake <u>O</u>f <u>C</u>ontradiction (FTSOC)
  - Attempt to prove that KB ∧ ¬*Query* ⊢ ⊥

Resolution Inference Rule

Basic propositional version:

$$\frac{\alpha \vee \beta, \quad \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or equivalently} \quad \frac{\neg\alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

Full first-order version:

$$\frac{\begin{array}{c} p_1 \vee \ldots \; p_j \; \ldots \vee p_m, \\ q_1 \vee \ldots \; q_k \; \ldots \vee q_n \end{array}}{(p_1 \vee \ldots \; p_{j-1} \vee p_{j+1} \; \ldots p_m \vee q_1 \ldots \; q_{k-1} \vee q_{k+1} \; \ldots \vee q_n)\sigma}$$

where $p_j\sigma = \neg q_k\sigma$

For example,

$$\frac{\begin{array}{l} \neg Rich(x) \vee Unhappy(x) \\ Rich(Me) \end{array}}{Unhappy(Me)}$$

with $\sigma = \{x/Me\}$

Adapted from slides by S. Russell, UC Berkeley

Digression: Decidability and Formal Languages

- See: Hopcroft and Ullman *2e*, Lewis and Papadimitriou *3e*
- Formal Languages (See: CIS 540, Other Automata Theory Course)
  - Member of Turing hierarchy
    - Finite state automata: regular languages
    - Pushdown automata: context-free languages
    - Linear bounded automata: context-sensitive languages
    - Turing machines: recursive languages
  - Recursive languages
    - ∃ computational model for decision problem, halts in finite number of steps
    - REC: set of all recursive languages
    - Example: finite searches (convert to decision problem: *checking solution*)
    - *Closed under complementation* (consequence?)
  - Recursive enumerable but not recursive (RE - REC)
  - Not recursive ($\notin$ RE)
- What Are FOL-VALID, FOL-NOT-SAT, FOL-SAT, FOL-NOT-VALID?

Universal quantification

- ∀*<variables> <sentence>*
- Everyone at K-State is smart:
- ∀x At(x, K-State) ⇒ Smart(x)
- ∀x *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of *P*
  - At(KingJohn, K-State) ⇒ Smart(KingJohn)
  - ∧ At(Richard, K-State) ⇒ Smart(Richard)
  - ∧ At(K-State, K-State) ⇒ Smart(K-State)
  - ∧ ...

A common mistake to avoid

- **Typically, ⇒ is the main connective with ∀**
- **Common mistake: using ∧ as the main connective with ∀:**

  ∀x At(x, K-State) ∧ Smart(x)

  means “Everyone is at K-State and everyone is smart”

Existential quantification

- **∃<*variables*> <*sentence*>**

- **Someone at KU is smart:**
- **∃*x* At(x, KU) ∧ Smart(x)$**

- **∃*x P* is true in a model *m* iff *P* is true with *x* being some possible object in the model**

- **Roughly speaking, equivalent to the disjunction of instantiations of *P***

  At(KingJohn, KU) ∧ Smart(KingJohn)

  ∨ At(Richard, KU) ∧ Smart(Richard)

  ∨ At(KU, KU) ∧ Smart(KU)

  ∨ ...

Summary Points

- Applications of Knowledge Bases (KBs) and Inference Systems
- "Industrial Strength" KBs
  - Building KBs
  - Components
    - Ontologies
    - Fact and rule bases
    - Knowledge Engineering (KE) and protocol analysis
    - Inductive Logic Programming (ILP) and other machine learning techniques
  - Using KBs
- Systems of Sequent Rules: GMP/AI/UE, Resolution
- Methodology of Inference
  - Inference as search
  - Forward and backward chaining
  - Fan-in, fan-out

Terminology

- Logical Frameworks
  - Knowledge Bases (KB)
  - Logic in general: representation languages, syntax, semantics
  - Propositional logic
  - First-order logic (FOL, FOPC)
  - Model theory, domain theory: possible worlds semantics, entailment
- Normal Forms
  - Conjunctive Normal Form (CNF)
  - Disjunctive Normal Form (DNF)
  - Horn Form
- Proof Theory and Inference Systems
  - Sequent calculi: rules of proof theory
  - Derivability or provability
  - Properties
    - Soundness (derivability implies entailment)
    - Completeness (entailment implies derivability)