



## Lecture 27 of 42

### Uncertain Reasoning: Probability Review Discussion: Probabilistic Reasoning Apps

Friday, 26 October 2007

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KSOL course page: <http://snipurl.com/v9v3>

Course web site: <http://www.kddresearch.org/Courses/Fall-2007/CIS730>

Instructor home page: <http://www.cis.ksu.edu/~bhsu>

Reading for Next Class:

Chapter 13, Russell & Norvig 2<sup>nd</sup> edition



Min	85
Max	189
Mean	147
Median	158
Stdev	30.6

<u>850</u>	A
<u>700</u>	B
<u>550</u>	C



## Lecture Outline

- Today's Reading: Sections 12.5 – 12.8, R&N 2e
- Next Week's Reading: Chapter 13, Sections 14.1 – 14.2, R&N 2e
- Today: Intro to Uncertain Reasoning
  - \* Nondeterminism in the real world
  - \* Incomplete domain theories
  - \* Observation errors: sensor, measurement, estimation
  - \* Actuator errors
- Probability Review
  - \* Kolmogorov axioms
  - \* Conditioning
- Next Week: Graphical models
  - \* Bayesian networks and causality
  - \* Inference and learning
  - \* BNJ interface (<http://bnj.sourceforge.net>)



## Looking Ahead [1]: Planning and Learning Roadmap

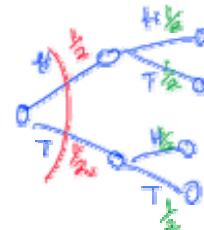
- Bounded Indeterminacy (12.3)
- Four Techniques for Dealing with Nondeterministic Domains
- 1. Sensorless aka Conformant Planning: “Be Prepared” (12.3)
  - \* Idea: be able to respond to any situation (universal planning)
  - \* Coercion
- 2. Conditional aka Contingency Planning: “Review the Situation” (12.4)
  - \* Idea: be able to respond to many typical alternative situations
  - \* Actions for sensing
- 3. Execution Monitoring and Replanning: “The Show Must Go On” (12.5)
  - \* Idea: be able to resume momentarily failed plans
  - \* Plan revision
- 4. Continuous Planning: “Always in Motion, The Future Is” (12.6)
  - \* Lifetime planning (and learning!)
  - \* Formulate new goals





## Probability: Basic Definitions and Axioms

- **Sample Space ( $\Omega$ ):** Range of a Random Variable  $X$
- **Probability Measure  $Pr(\bullet)$** 
  - \*  $\Omega$  denotes a range of “events”;  $X: \Omega$
  - \* **Probability  $Pr$ , or  $P$ , is a measure over  $2^\Omega$**
  - \* In a general sense,  $Pr(X = x \in \Omega)$  is a measure of **belief** in  $X = x$ 
    - ⇒  $P(X = x) = 0$  or  $P(X = x) = 1$ : **plain** (aka **categorical**) beliefs (can't be revised)
    - ⇒ All other beliefs are subject to **revision**
- **Kolmogorov Axioms**
  - \* 1.  $\forall x \in \Omega . 0 \leq P(X = x) \leq 1$
  - \* 2.  $P(\Omega) \equiv \sum_{x \in \Omega} P(X = x) = 1$
  - \* 3.  $\forall X_1, X_2, \dots, \exists i \neq j \Rightarrow X_i \wedge X_j = \emptyset$ .
$$P\left(\bigcup_{i=1}^{\infty} X_i\right) = \sum_{i=1}^{\infty} P(X_i)$$
- **Joint Probability:**  $P(X_1 \wedge X_2) \equiv$  Probability of the Joint Event  $X_1 \wedge X_2$
- **Independence:**  $P(X_1 \wedge X_2) = P(X_1) \cdot P(X_2)$



## Basic Formulas for Probabilities

- **Product Rule (Alternative Statement of Bayes's Theorem)**

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

- \* **Proof:** requires axiomatic set theory, as does Bayes's Theorem

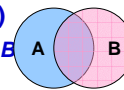
- **Sum Rule**

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

- \* **Sketch of proof (immediate from axiomatic set theory)**

⇒ Draw a Venn diagram of two sets denoting events  $A$  and  $B$

⇒ Let  $A \cup B$  denote the event corresponding to  $A \vee B$ ...



- **Theorem of Total Probability**

- \* Suppose events  $A_1, A_2, \dots, A_n$  are mutually exclusive and exhaustive

⇒ **Mutually exclusive:**  $i \neq j \Rightarrow A_i \wedge A_j = \emptyset$

⇒ **Exhaustive:**  $\sum P(A_i) = 1$

- \* Then  $P(B) = \sum_{i=1}^n P(B|A_i) \cdot P(A_i)$

- \* **Proof:** follows from product rule and 3<sup>rd</sup> Kolmogorov axiom



## Bayes's Theorem [1]

Product rule  $P(A \wedge B) = P(A|B)P(B) = P(B|A)P(A)$

$$\Rightarrow \text{Bayes' rule } P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Why is this useful???

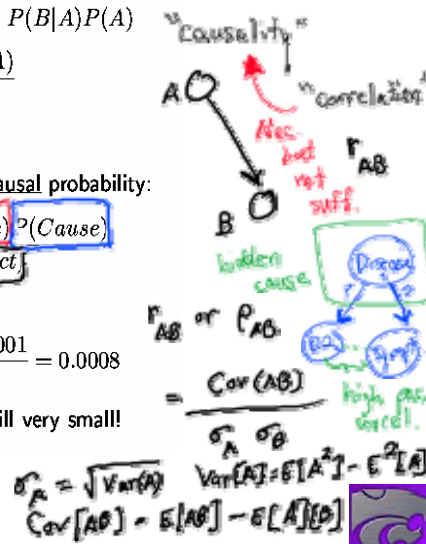
For assessing diagnostic probability from causal probability:

$$P(\text{Cause}|\text{Effect}) = \frac{P(\text{Effect}|\text{Cause})P(\text{Cause})}{P(\text{Effect})}$$

E.g., let  $M$  be meningitis,  $S$  be stiff neck:

$$P(M|S) = \frac{P(S|M)P(M)}{P(S)} = \frac{0.8 \times 0.0001}{0.1} = 0.0008$$

Note: posterior probability of meningitis still very small!



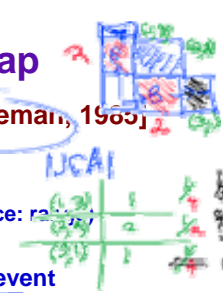
Adapted from slides by S. Russell, UC Berkeley



## Looking Ahead [2]: Uncertain Reasoning Roadmap

### ● Framework: Interpretations of Probability [Cheeseman, 1965]

- \* **Bayesian subjectivist view**
  - ⇒ Measure of an agent's belief in a proposition
  - ⇒ Proposition denoted by random variable (sample space:  $\Omega$ )
  - ⇒ e.g.,  $Pr(\text{Outlook} = \text{Sunny}) = 0.8$
- \* **Frequentist view: frequency of observations of an event**
- \* **Logicist view: inferential evidence in favor of proposition**



### ● Some Applications

- \* HCI: learning natural language; intelligent displays; decision support
- \* Approaches: prediction; sensor and data fusion (e.g., bioinformatics)

### ● Prediction: Examples

- \* Measure *relevant parameters*: temperature, barometric pressure, wind speed
- \* Make statement of the form  $Pr(\text{Tomorrow's-Weather} = \text{Rain}) = 0.5$
- \* College admissions:  $Pr(\text{Acceptance}) = p$ 
  - ⇒ Plain beliefs: unconditional acceptance ( $p = 1$ ), categorical rejection ( $p = 0$ )
  - ⇒ Conditional beliefs: depends on reviewer (use probabilistic model)



## Automated Reasoning using Probabilistic Models: Inference Tasks

Simple queries: compute posterior marginal  $P(X_i | \mathbf{E} = \mathbf{e})$   
 e.g.,  $P(\text{NoGas} | \text{Gauge} = \text{empty}, \text{Lights} = \text{on}, \text{Starts} = \text{false})$

Conjunctive queries:  $P(X_i, X_j | \mathbf{E} = \mathbf{e}) = P(X_i | \mathbf{E} = \mathbf{e})P(X_j | X_i, \mathbf{E} = \mathbf{e})$

Optimal decisions: decision networks include utility information;  
 probabilistic inference required for  $P(\text{outcome} | \text{action}, \text{evidence})$

Value of information: which evidence to seek next?

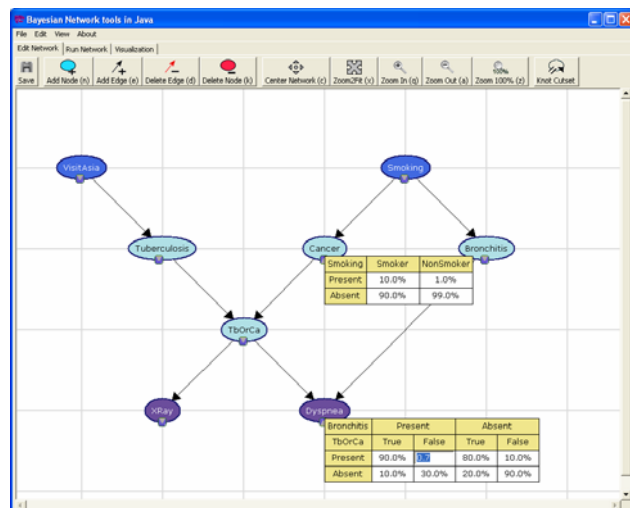
Sensitivity analysis: which probability values are most critical?

Explanation: why do I need a new starter motor?

Adapted from slides by S. Russell, UC Berkeley



## Looking Ahead [3]: Bayesian Network tools in Java (BNJ)

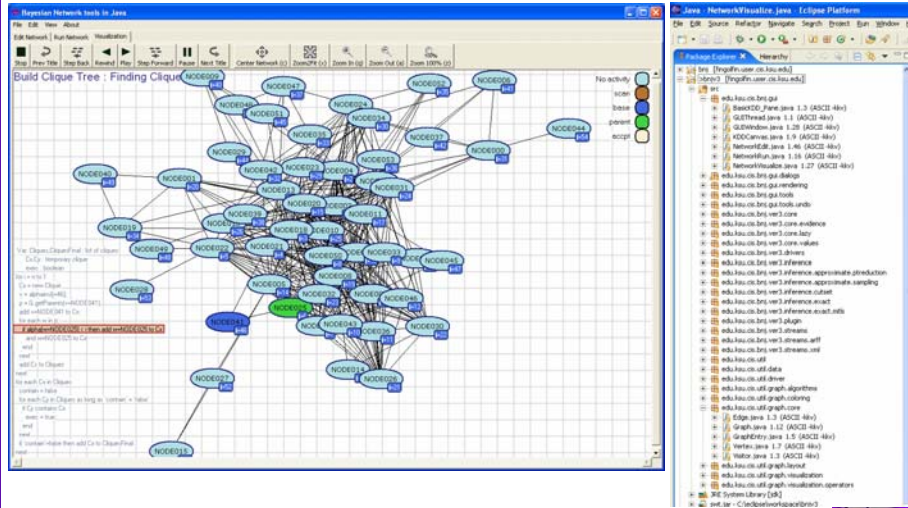


Asia (Chest Clinic) Network

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## BNJ Core [2] Graph Architecture



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CPCS-54 Network



## Bayes's Theorem [2]

- **Theorem**

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)} = \frac{P(h \wedge D)}{P(D)}$$

- $P(h) \equiv$  **Prior Probability of Assertion (Hypothesis)  $h$** 
  - \* Measures initial beliefs (BK) before any information is obtained (hence prior)
- $P(D) \equiv$  **Prior Probability of Data (Observations)  $D$** 
  - \* Measures probability of obtaining sample  $D$  (i.e., expresses  $D$ )
- $P(h | D) \equiv$  **Probability of  $h$  Given  $D$** 
  - \* | denotes conditioning - hence  $P(h | D)$  is a conditional (aka posterior) probability
- $P(D | h) \equiv$  **Probability of  $D$  Given  $h$** 
  - \* Measures probability of observing  $D$  given that  $h$  is correct (“generative” model)
- $P(h \wedge D) \equiv$  **Joint Probability of  $h$  and  $D$** 
  - \* Measures probability of observing  $D$  and of  $h$  being correct



## Bayesian Inference: Query Answering (QA)

- **Answering User Queries**

- \* Suppose we want to perform intelligent inferences over database *DB*
  - ⇒ Scenario 1: *DB* contains records (instances), some “labeled” with answers
  - ⇒ Scenario 2: *DB* contains probabilities (annotations) over propositions
- \* QA: an application of probabilistic inference

- **QA Using Prior and Conditional Probabilities: Example**

- \* Query: *Does patient have cancer or not?*
- \* Suppose: patient takes a lab test and result comes back positive
  - ⇒ Correct + result in only 98% of cases where disease actually present
  - ⇒ Correct - result in only 97% of cases where disease not present
  - ⇒ Only 0.008 of the entire population has this cancer
- \*  $\alpha \equiv P(\text{false negative for } H_0 \equiv \text{Cancer}) = 0.02$  (NB: for 1-point sample)
- \*  $\beta \equiv P(\text{false positive for } H_0 \equiv \text{Cancer}) = 0.03$  (NB: for 1-point sample)

$$P(\text{Cancer}) = 0.008 \quad P(+ | \text{Cancer}) = 0.98 \quad P(+ | \neg \text{Cancer}) = 0.03$$

$$P(\neg \text{Cancer}) = 0.992 \quad P(- | \text{Cancer}) = 0.02 \quad P(- | \neg \text{Cancer}) = 0.97$$

- \*  $P(+ | H_0) P(H_0) = 0.0078$ ,  $P(+ | H_A) P(H_A) = 0.0298 \Rightarrow h_{MAP} = H_A \equiv \neg \text{Cancer}$



## Choosing Hypotheses

- **Bayes's Theorem**

$$P(h | D) = \frac{P(D | h)P(h)}{P(D)} = \frac{P(h \wedge D)}{P(D)}$$

- **MAP Hypothesis**

- \* Generally want most probable hypothesis given the training data
- \* Define:  $\arg \max_{x \in \Omega} [f(x)]$   $\equiv$  the value of  $x$  in the sample space  $\Omega$  with the highest  $f(x)$
- \* Maximum a posteriori hypothesis,  $h_{MAP}$

$$h_{MAP} = \arg \max_{h \in H} P(h | D)$$

$$= \arg \max_{h \in H} \frac{P(D | h)P(h)}{P(D)}$$

$$= \arg \max_{h \in H} P(D | h)P(h)$$

- **ML Hypothesis**

- \* Assume that  $p(h_i) = p(h_j)$  for all pairs  $i, j$  (uniform priors, i.e.,  $P_H \sim \text{Uniform}$ )
- \* Can further simplify and choose the maximum likelihood hypothesis,  $h_{ML}$

$$h_{ML} = \arg \max_{h_i \in H} P(D | h_i)$$



## Automated Reasoning using Probabilistic Models: Inference Tasks

Simple queries: compute posterior marginal  $P(X_i | \mathbf{E} = e)$

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$$\begin{aligned} h_{MAP} &= \arg \max_{h \in H} P(h | D) \\ &= \arg \max_{h \in H} \frac{P(D | h)P(h)}{P(D)} \\ &= \arg \max_{h \in H} P(D | h)P(h) \end{aligned}$$

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- \* Can further simplify and choose the maximum likelihood hypothesis,  
 $h_{ML}$

$$h_{ML} = \arg \max_{h_i \in H} P(D | h_i)$$





## Graphical Models of Probability

- **Conditional Independence**

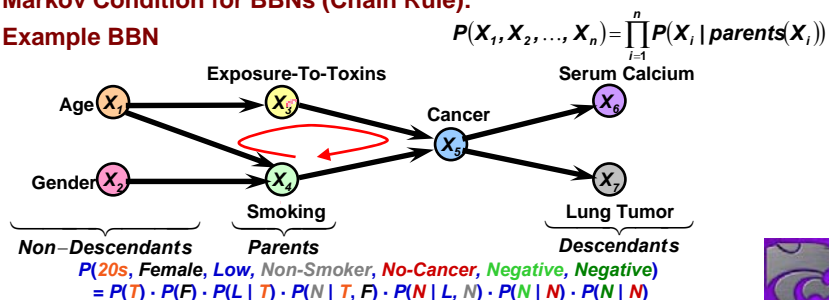
- \*  $X$  is **conditionally independent (CI)** from  $Y$  given  $Z$  iff  $P(X | Y, Z) = P(X | Z)$  for all values of  $X, Y,$  and  $Z$
- \* Example:  $P(\text{Thunder} | \text{Rain}, \text{Lightning}) = P(\text{Thunder} | \text{Lightning}) \Leftrightarrow T \perp R | L$

- **Bayesian (Belief) Network**

- \* **Acyclic directed graph model**  $B = (V, E, \Theta)$  representing **CI assertions** over  $\Theta$
- \* **Vertices (nodes)  $V$** : denote events (each a random variable)
- \* **Edges (arcs, links)  $E$** : denote conditional dependencies

- **Markov Condition for BBNs (Chain Rule):**

- **Example BBN**



## Semantics of Bayesian Networks

“Global” semantics defines the full joint distribution as the product of the local conditional distributions:

$$P(X_1, \dots, X_n) = \prod_{i=1}^n P(X_i | \text{Parents}(X_i))$$

e.g.,  $P(J \wedge M \wedge A \wedge \neg B \wedge \neg E)$  is given by??

$$= P(\neg B)P(\neg E)P(A|\neg B \wedge \neg E)P(J|A)P(M|A)$$

“Local” semantics: each node is conditionally independent of its nondescendants given its parents

Theorem: Local semantics  $\Leftrightarrow$  global semantics

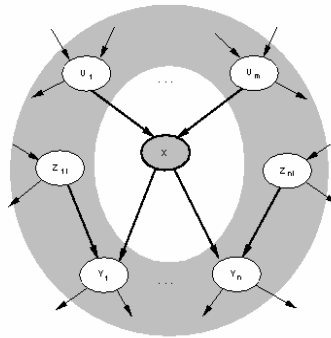


Adapted from slides by S. Russell, UC Berkeley



## Markov Blanket

Each node is conditionally independent of all others given its Markov blanket: parents + children + children's parents



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## Constructing Bayesian Networks: The Chain Rule of Inference

Need a method such that a series of locally testable assertions of conditional independence guarantees the required global semantics

1. Choose an ordering of variables  $X_1, \dots, X_n$

2. For  $i = 1$  to  $n$

add  $X_i$  to the network

select parents from  $X_1, \dots, X_{i-1}$  such that

$$\mathbf{P}(X_i | \text{Parents}(X_i)) = \mathbf{P}(X_i | X_1, \dots, X_{i-1})$$

This choice of parents guarantees the global semantics:

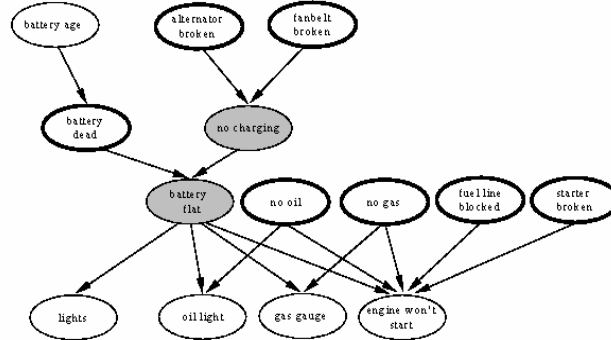
$$\begin{aligned} \mathbf{P}(X_1, \dots, X_n) &= \prod_{i=1}^n \mathbf{P}(X_i | X_1, \dots, X_{i-1}) \text{ (chain rule)} \\ &= \prod_{i=1}^n \mathbf{P}(X_i | \text{Parents}(X_i)) \text{ by construction} \end{aligned}$$

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## Example: Evidential Reasoning for Car Diagnosis

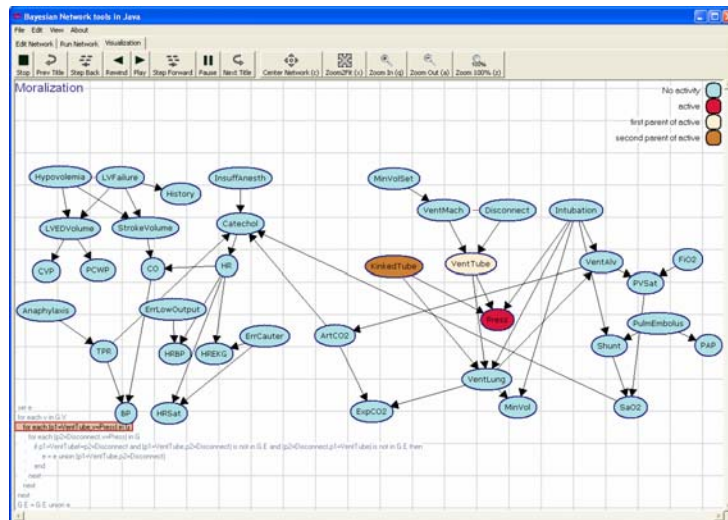
Initial evidence: engine won't start  
 Testable variables (thin ovals), diagnosis variables (thick ovals)  
 Hidden variables (shaded) ensure sparse structure, reduce parameters



Adapted from slides by S. Russell, UC Berkeley



## BNJ Visualization [2] Pseudo-Code Annotation (Code Page)



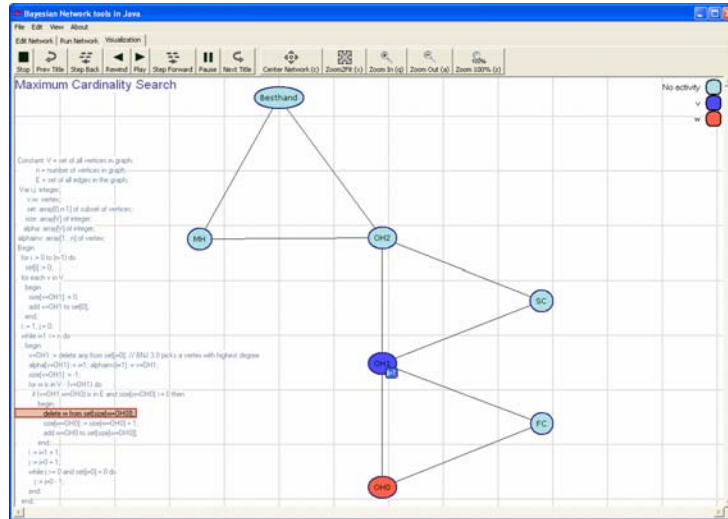
ALARM  
Network

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## BNJ Visualization [3] Network



Poker  
Network

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## Terminology

- **Introduction to Reasoning under Uncertainty**
  - \* Probability foundations
  - \* Definitions: subjectivist, frequentist, logistic
  - \* (3) Kolmogorov axioms
- **Bayes's Theorem**
  - \* Prior probability of an event
  - \* Joint probability of an event
  - \* Conditional (posterior) probability of an event
- **Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Hypotheses**
  - \* MAP hypothesis: highest conditional probability given observations (data)
  - \* ML: highest likelihood of generating the observed data
  - \* ML estimation (MLE): estimating parameters to find ML hypothesis
- **Bayesian Inference: Computing Conditional Probabilities (CPs) in A Model**
- **Bayesian Learning: Searching Model (Hypothesis) Space using CPs**



## Summary Points

- Introduction to Probabilistic Reasoning
  - \* Framework: using probabilistic criteria to search  $H$
  - \* Probability foundations
    - ⇒ Definitions: subjectivist, objectivist; Bayesian, frequentist, logicist
    - ⇒ Kolmogorov axioms
- Bayes's Theorem
  - \* Definition of conditional (posterior) probability
  - \* Product rule
- Maximum A Posteriori (MAP) and Maximum Likelihood (ML) Hypotheses
  - \* Bayes's Rule and MAP
  - \* Uniform priors: allow use of MLE to generate MAP hypotheses
  - \* Relation to version spaces, candidate elimination
- Next Week: Chapter 14, Russell and Norvig
  - \* Later: Bayesian learning: MDL, BOC, Gibbs, Simple (Naïve) Bayes
  - \* Categorizing text and documents, other applications

